# Beam Concepts for Future Muon g-2 Experiments
*not for distribution*

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# 1 Motivation and Options for Beamline Upgrade

The E821 experiment [1] measured the anomalous muon magnetic moment $a_\mu$ to 0.54 ppm. The current theoretical uncertainty is 0.48 ppm. The difference $a_\mu(exp) - a_\mu(theory) = (27.6 \pm 8.4) \times 10^{-10}$ represents a 3.3 $\sigma$ discrepancy between experiment and the Standard Model prediction [2]. Experiment E969 [3] at BNL has been proposed to decrease the experimental uncertainty by a factor of 2.5. In order to achieve such an improvement at least a factor of 5 times higher muon flux in the g-2 ring is required. Using the E696 proposal as a benchmark, we consider several options for advanced beamlines to fulfil the muon beam requirements for future g-2 experiments.

## 1.1 Existing Beamline

The E821 beamline is comprised of five sections: the pion collection and pion spectrometer section, the matching section, the decay channel, the muon spectrometer, and the matching section to the storage ring. A dispersed focus (1 cm/%) is produced by Q1Q2D1D2Q3Q4 at the slits K1K2, and the dispersion is removed by Q5Q6D3D4. Q7 through Q10 rotate the beam envelope of both planes to produce a double waist at the center of Q11. Q11 through Q19 are a simple FODO channel with a phase advance per cell of 90°. D5Q20Q21Q22 produce a dispersive focus at the slits K3K4 just upstream of Q23, and Q23Q24Q25D6 remove the dispersion. Finally, Q26Q27Q28Q29 reform the beam envelope for passage through the inflector and then into the storage ring. These optical constraints determine an essentially unique tune for the beamline. From reference [1] we quote some relevant results characterizing the current beam line in Tables 1,2,3.

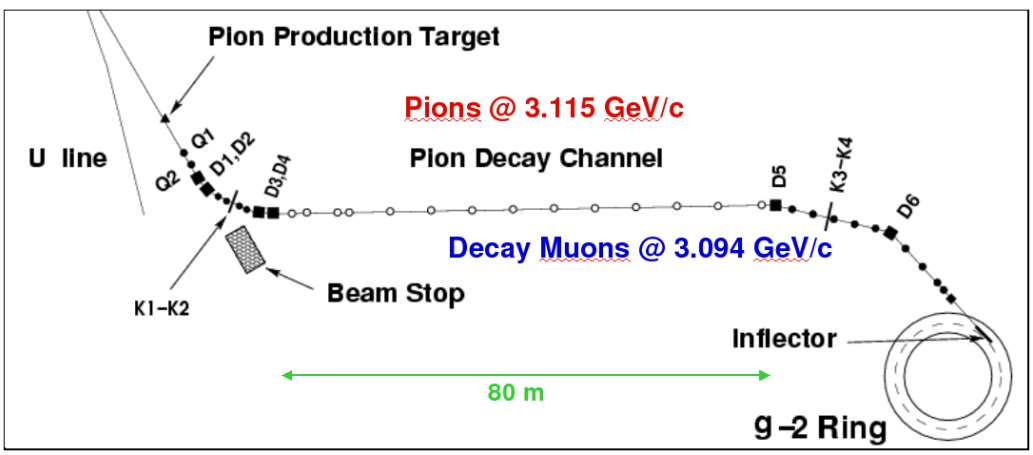



Figure 1: Existing E821 beamline at BNL

| Proton Beam | Value | Pion Beamline | Value |
|---|---|---|---|
| Protons per AGS cycle | $5 \times 10^{13}$ | Pions per proton* | $10^{-5}$ |
| Cycle repetition rate | 0.37 Hz | Muons per pion decay** | 0.012 |
| Proton momentum | 24 GeV/$c$ | | |
| Bunches per cycle | 6 to 12 | | |
| Bunch width ($\sigma$) | 25 ns | | |
| Bunch spacing | 33 ms | | |

Table 1: From Ref. [1]: Selected AGS proton beam and secondary pion beamline characteristics
*Captured by the beamline channel; **Measured at the inflector entrance

| Section | horizonal acceptance ($\pi$ $mm - mrad$) | aperture (mm) | angle (mrad) | vertical acceptance ($\pi$ $mm - mrad$) | aperture (mm) | angle (mrad) |
|---|---|---|---|---|---|---|
| Injection | 42 | | $\pm 32$ | 56 | | $\pm 60$ |
| FODO | | | | | | |
| Inflector | 42 (48) | $\pm 9$ | $\pm 5.3$ | 462 | $\pm 28$ | $\pm 16.5$ |
| Ring ($\delta p = 0$) | 265 | $\pm 45$ | $\pm 5.9$ | 105 | $\pm 45$ | $\pm 2.3$ |
| Infl.& Ring | 42 (48) | $\pm 9$ | $\pm 5.3$ | 64 | $\pm 28$ | $\pm 2.3$ |

Table 2: Acceptances for E821 experiment. Geometrical acceptance estimates from Bill Morse (Peter Kammel) estimates.

| $p_\pi/p_\mu$ | $N_\mu$(Calc)* | $N_\mu$(Meas) | $F_\pi$(Calc) | $F_\pi$(Meas) | $P_\mu$(Calc) | $A$(Meas) | rel. FOM |
|---|---|---|---|---|---|---|---|
| 1.005 | 1 | 1 | 0.78 | 0.80 | 0.99 | 0.22 | 1.00 |
| 1.010 | 0.5 | 0.43 | 0.29 | 0.30 | 0.98 | 0.26 | 0.60 |
| 1.015 | 0.29 | 0.21 | 0.04 | 0.065 | 0.96 | 0.30 | 0.39 |
| 1.017 | 0.25 | 0.17 | 0.002 | 0.016 | 0.96 | 0.30 | 0.32 |
| 1.020 | 0.18 | 0.12 | - | 0.009 | 0.95 | 0.30 | 0.22 |

Table 3: Modified from Ref. [1]: As a function of the ratio of central-pion to muon momentum $p_\pi/p_\mu$: From left to right, calculated relative muon transmission fraction; measured relative stored muon flux; calculated and measured pion transmission fraction into the ring; calculated muon polarization; measured asymmetry $A$ using $E_{th} = 1.5$ GeV; figure-of-merit (FOM). The absolute fraction of muons per pion decay is obtained by multiplying column 2 by 0.018. The FOM is calculated as the measured $N_\mu \times A^2$. *We don't agree, but calculate a factor of 2 between $p_\pi/p_\mu$=1.005 and 1.017

## 1.2 Beam Options Overview

### 1.2.1 Forward Decay Beam

This is the conservative baseline approach. We assume that a change of $p_\pi = 1.03\ p_\mu$ is required to keep the hadronic flash under control. Fig 2 represents our current best, realistic forward decay beam design with a 4x FODO lattice. The estimated performance is summarized in Tab. 4.

| Improvement | N | B | R | Comment |
|---|---|---|---|---|
| AGS bunch increase | 1 | 1 | R | Attractive possibility, but uncertain |
| 4x FODO quads | 2.5 | 0.33 | 2.5 | Studies of $p_\pi/p_\mu = 1.03$ |
| Open inflector ends | 2 | 1 | 2 | Preliminary studies |
| Kicker | 2 | 1 | 2 | Simulation and engineering required |

Table 4: Modified from Ref. [2]: E969 baseline improvements compared to E812. N, B and R represent the overall stored muon FOM, the relative hadron induced background, and the initial instantaneous rate, respectively.

### 1.2.2 Backward Decay Beam with Achromat

The backward decay beam eliminates the flash. Probably the bend in the pion injection has to be replaced by an achromat so that the concept benefits from the wider allowed $\Delta p_\pi$ band contributing to magic muons. A schematic beamline design is shown in fig. 3. At the moment the challenge is the simultaneous reduction of second order chromatic effects from both the pion injection and the bend.

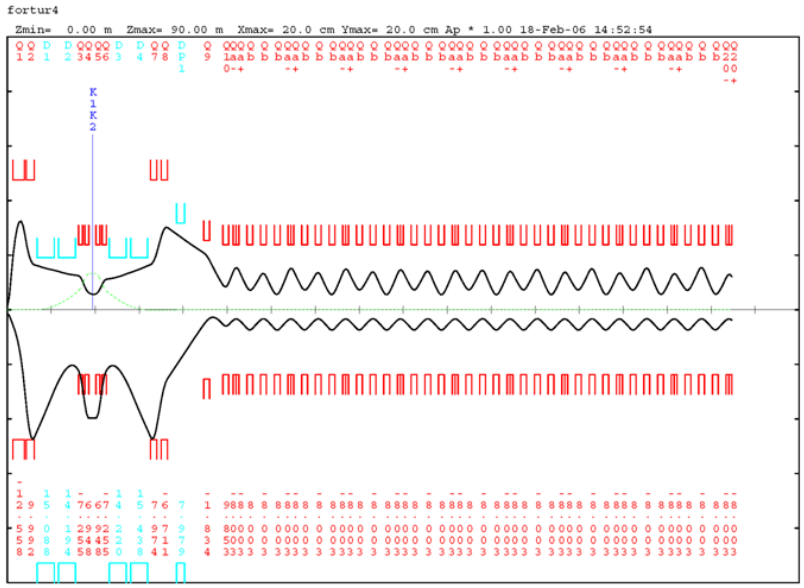



Figure 2: Lattice design for E969 forward decay beam

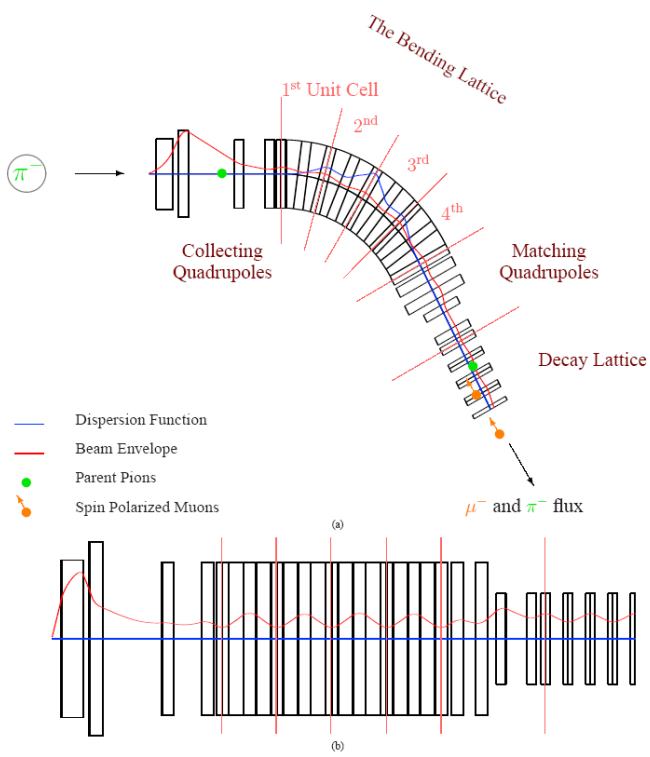



Figure 3: Sketch of Achromat from Sara's prelim.

### 1.2.3 Muon Accumulator Ring

The Muon Accumulator Ring (MAR) is sketched in fig. 4. Its potential lies in the following capabilities

| capability | gain factor compared to E821 |
|---|---|
| longer decay length | $\approx 2$ |
| zero degree forward decay for $p_\pi = 1.005\ p_\mu$ | $\approx 2$ |
| integration over larger $\Delta p_\pi$ band | $\approx 1.5$ |
| FODO 2x | $\approx 2$ |
| total | $\approx 12$ |

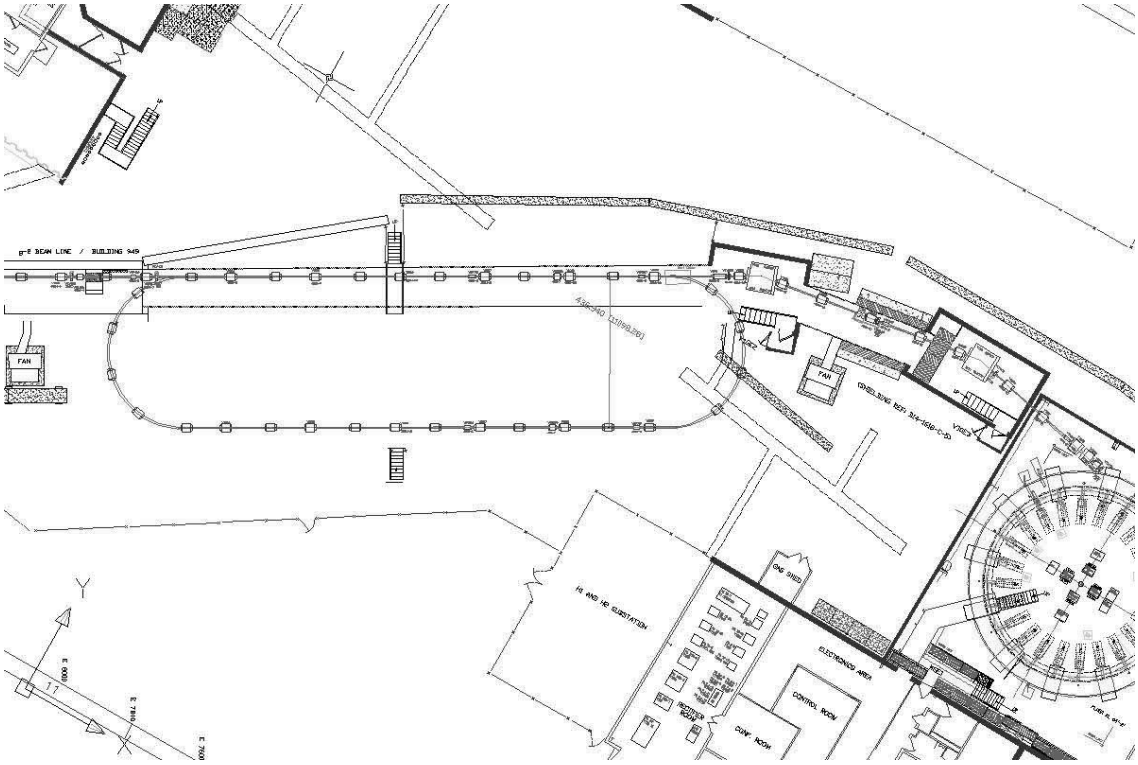



Figure 4: Sketch of Muon Accumulator Ring potential location at BNL.

A first lattice design is presented later in this paper. Issues to be carefully addressed include i) quantitative pion suppression, ii) debunching, iii) potential polarization losses due to dephasing. A general potential problem, which probably applies to all scenarios to different extent, it the polarization to phase space correlation. This might lead to polarization dependent loss terms in the ring and thus to a potentially dangerous polarization phase shift.

### 1.2.4 Fermilab 8 GeV plans

Fermilab is considering a plan to reconfigure its accelerator complex into an intense proton source, which simultaneously would support a broad experimental program. Of main interest for the g-2 experiment is the 8 GeV experimental facility with a flux of 22.7 $10^{16}$ p/h from the Fermilab booster. The booster repetition rate would gradually be increased from 9 to 13.5 Hz. The extracted intensity from the booster would be 4.1-4.7 $10^{12}$ protons, comparable to the bunch intensity of $5 \times 10^{13}/12 = 4.2 \times 10^{12}$ protons from the AGS. E821 was running with an flux of 6.6 $10^{16}$ p/h (Table 1). Issues to be studied include i) 3.1 GeV pion production comparison for 24 and 8 GeV protons, ii) rebunching of the 1.6 $\mu$s booster pulse to 150ns (g-2 ring cycle time) and iii) study whether the 8 GeV complex can be used as muon accumulator ring.

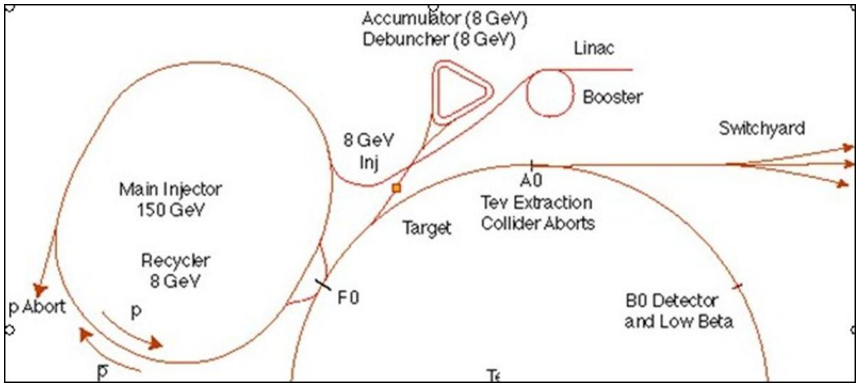



Figure 5: FNAL complex: After run 2 is finished all three 8 GeV rings will become available for the neutrino program at the main injector and an 8 GeV physics program.

## 2 Definition of Figure of Merit

In principle, the FOM is easy to define. Let us define $NP^2 =< NP^2 >$ for the magic muons stored in the ring and the flash ratio $R_F = A(pedestal)/A(e@2GeV)$ as the ratio between the pedestal amplitude and the amplitude of a 2 GeV decay electron in the electromagnetic calorimeter at time T=30 $\mu$s after muon injection in the ring.

The FOM will be a compromise between large $NP^2$ and small $R_F$ [1]. At the moment we have not performed studies to quantify the importance of $R_F$ and it does not show up directly in the error table of Ref. [1]. Such a study has to be done and documented, before we can be more specific.

In practice, however, the definition of a FOM is much more complex. Both $NP^2$ and $R_F$ depend on the interplay of the whole transport system responsible for delivering the beam into the g-2 ring. Information about some of those components are available from E821 measurements, but for the majority of components, in particular new upgrade ideas, one relies on simulations. Future tests could help in better defining the performance of new components and to improve the relevance and reliability of existing measurements. At the moment, the predicted FOM has to rely on a combination of simulation and experimental information, where one tries to minimize uncertainties with relative comparisons and by avoiding information where existing experimental data and simulation disagree.

The main parameters characterizing the beam are the transverse phase space coordinate (x,x',y,y'), the relative momentum deviation $\delta p$ and the longitudinal path length difference $\delta l$. We define the overall width of the pulse as W. For the discussion we break the beam transport system into separate parts.

1. **Beamline and muon collector**. This is the secondary $\pi - \mu$ beam transport and muon collection system from the production target to the inflector entrance.

2. **Inflector**. This inflector transports the beam into the ring. The influence of open and closed ends on the phase space has to be studied.

3. **Kicker and ring**. The kicker deflects the entering beam on the central orbit of the ring. It is critical to understand the kicker's performance, efficiency and potential improvements.

Clearly all systems are strongly interlinked. For instance notice that the optimal pulse length W critically depends on the width of the kicker pulse.

At this point we don't have clear acceptance functions for inflector and kicker and ring. Thus we define the FOM of the beamline in the following preliminary way.

- The forward and backward beam lines use the magnets described in the turtle input files in the appendix. For the backward beam, part of the pion injection magnets have to be replaced. Those have been modeled according to Phil's prescription in the proposal [3]. The forward beam assumes asymmetric K1/K2 settings 1200/600, respectively. This cuts $p_\pi$ more strongly at low

[1] How does $R_F$ depend on the calorimeter's granularity?

momenta, which is key for flash suppression. For the forward beam the calculated intensity is reduced by 8% corresponding to the average rate over the production target length of 192 mm.

- The properties of the magic muons are studied at the center of the last FODO magnet (Q20 in the original E821 beamline). From there it is assumed that a momentum bite of $\delta p_\mu = 0.25\%$ is accepted by the g-2 ring. The effect of the slits K3/K4 are ignored, as it is assumed that they serve only as a flash suppression and don't cut into the magic muons within the ring acceptance. It is further assumed that the beam optics to the inflector can be tuned to optimize the matching of the simulated emittance of a particular tune $\epsilon_x$ and $\epsilon_y$ to the combined inflector-ring acceptance of $\Gamma_x = 48\pi mm - mrad$ and $\Gamma_y = 64\pi mm - mrad$ (c.f. Table 2) [2].

The final figure of merit FOM is defined as

$$FOM = N_\mu < P^2 > p(\epsilon_x)p(\epsilon_y), \tag{1}$$

where p($\epsilon_x$) and p($\epsilon_y$) are the probabilities that the magic muons are within the horizontal and vertical inflector-ring acceptance, respectively. The probability that two Gaussian distributed random variables (x, x') lie simultaneously within an ellipsoid of constant probability is given by the $\chi^2(x, n = 2)$ distribution. Specifically,

$$p(\epsilon_x) = 1 - \int_C^\infty \chi^2(x, n = 2)dx \tag{2}$$

where $C = \Gamma_x/\epsilon_x$. A standard Excel library function for the cumulative $\chi^2$ distribution was used. The y phase space acceptance factor has an analogous form.

Obviously, the inflector, kicker, ring part of the muon transport is taken into account only schematically, as it has not been sufficiently modeled up to now. The somewhat indirect definition of the acceptance factors p($\epsilon$) is a result of this shortcoming, and could be completely eliminated by consistently modeling the beam transport into the ring. This should be done. The pathlength difference $\delta l$ is also ignored.

Paul wants to mention the problem of dispersion mismatch in the ring?

# 3 Design and Simulations

## 3.1 Forward Backward Comparison

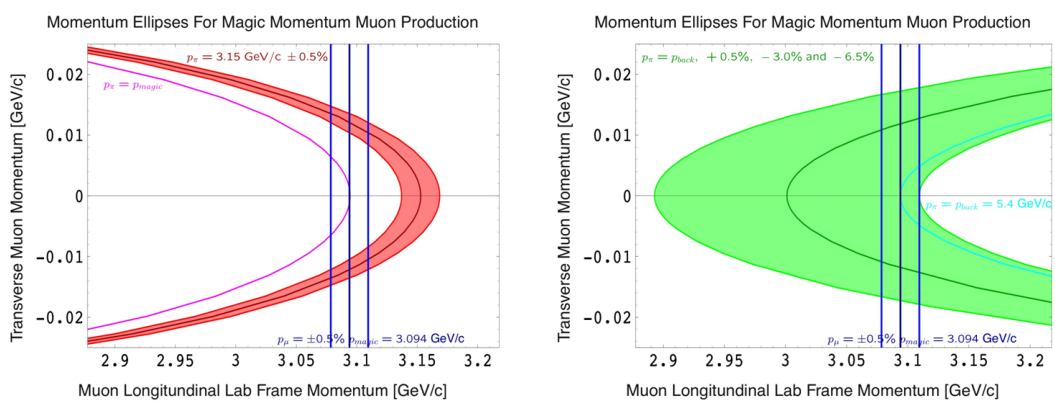



Figure 6: Forward-backward decay kinematics

The difference in kinematics between forward and backward decay is illustrated in Fig. 6. A simple analytical comparison is presented in Table 3.1. It does not seems possible to favor one solution over the other, since several factors balance each other. Thus a detailed beam simulation with TURTLE is required.

| quantity | forward | backward | comment |
|---|---|---|---|
| $p_\pi$ | 3.152 | 5.22 | (GeV/c) |
| $\gamma$ | 22.6 | 37.41 | |
| $\frac{\partial N}{\partial p_L} \propto \frac{1}{\gamma}$ | | | $p_L$ muon longitudinal $p_{LAB}$ |
| $\frac{\partial N}{\partial \theta} \propto \gamma$ | | | $\theta$ muon polar angle in LAB |
| Polarization | higher for fixed $\theta$ | smaller $\theta$ accepted | |
| $\delta p$ pion | $\approx 0.5\%$ & $\theta = 4mrad$ | $\approx 3\%$ & $\theta = 0 - 8mrad$ | |
| pion decay in FODO | 0.382 | 0.254 | |
| pion production rate | 1 | 1.1 | |

Table 5: Analytical comparison of factors determining forward and backward flux and FOM

**Fitted RMS**

| | | | | | | | | | | | Sigma within admittance | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Condition | N | X | Y | P | sig | Ncorr[2] | Norm N | ABS(P) | NP2 | Norm NP2 | XX' | YY' | within XX' | within YY' | FOM | Norm FOM | |
| for 1x | 3406 | 3.18 | 4.22 | -0.945 | 0.014 | 3134 | 1.000 | 0.945 | 0.893 | 1.000 | 1.51 | 1.52 | 0.53 | 0.53 | 0.28 | 1 | for1kg |
| for 2x | 8524 | 3.41 | 4.51 | -0.943 | 0.014 | 7842 | 2.503 | 0.943 | 2.226 | 2.492 | 1.41 | 1.42 | 0.51 | 0.51 | 0.64 | 2.27 | for2kg |
| for 4x | 12674 | 3.26 | 5.48 | -0.942 | 0.015 | 11660 | 3.721 | 0.942 | 3.303 | 3.698 | 1.47 | 1.17 | 0.52 | 0.44 | 0.85 | 3.03 | for4kg |
| for 4x pi+3% | 12019 | 3.82 | 6.01 | -0.909 | 0.015 | 11057 | 3.529 | 0.909 | 2.917 | 3.265 | 1.26 | 1.06 | 0.47 | 0.41 | 0.63 | 2.23 | for4kp12g |
| for 4x pi+5% | 11049 | 4.74 | 6.54 | -0.854 | 0.015 | 10165 | 3.244 | 0.854 | 2.367 | 2.650 | 1.01 | 0.98 | 0.40 | 0.39 | 0.41 | 1.45 | for4kp32g |
| back 1x | 2070 | 2.44 | 2.81 | 0.948 | 0.034 | 2070 | 0.661 | 0.948 | 0.594 | 0.666 | 1.97 | 2.28 | 0.63 | 0.68 | 0.28 | 1.01 | back1g |
| back 2x | 6028 | 3.62 | 4.65 | 0.911 | 0.049 | 6028 | 1.924 | 0.911 | 1.601 | 1.793 | 1.33 | 1.38 | 0.48 | 0.50 | 0.43 | 1.53 | back2m10g |
| back 4x | 8470 | 4.03 | 6.16 | 0.9 | 0.053 | 8470 | 2.703 | 0.900 | 2.197 | 2.460 | 1.19 | 1.04 | 0.45 | 0.41 | 0.45 | 1.59 | back4m13g |
| back 4x qs | 9433 | 3.98 | 6.85 | 0.892 | 0.06 | 9433 | 3.010 | 0.892 | 2.406 | 2.694 | 1.21 | 0.93 | 0.45 | 0.37 | 0.46 | 1.62 | back4qsm13g |

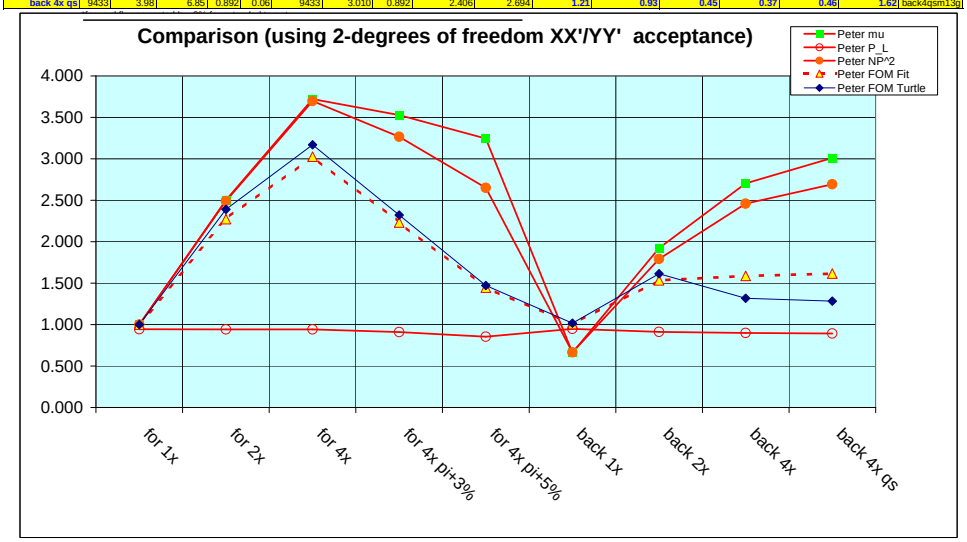



Figure 7: Forward-backward normalized FOM decay turtle results

The numerical results for turtle simulation with $5 \times 10^7$ events are summarized in Fig. 7. As regards the forward beam the $p_\pi/p_\mu = 1.03$ point is promising. It results in a FOM $\approx 2.23$ compared to E821. The flash should be further reduced by increasing $p_\pi/p_\mu$ from nominal 1.017 to 1.03. The exact amount could be determined in a beam test. The backward beam gets a big hit from the acceptance matching to the inflector/ring. With 4x FODO the increase in muon rate does not increase the FOM. In addition, the backdecay suffers from the fact, that, due to second order chromatic effects, the channel does not transport a large pion momentum band, which could contribute to magic muons. Thus only a factor of $\approx 1.59$ FOM improvement relative to E821 is calculated. The correct way of including the p($\epsilon$) is critical and needs to be studied thoroughly by a continuing the simulation up into the g-2 ring.

## 3.2 Second Order Achromat Design

This section consists of new calculations by Sara, which have not been checked independently. Paul, do you want to check this?.

[2] As the TURTLE rms are slightly larger than the Gaussian fit to the TURTLE marginal distributions in each phase space variable (x, x', y, y'), we use the emittance based on the Gaussian fit, arguing that better describes the bulk of the beam.

### 3.2.1 Schematic Design

Sara designed an achromat following Karl Brown's prescription [5]. The details are presented in an appendix. Briefly, the design concept consists of 2 steps. i) A lattice made of 4 identical cells allows the elimination of all 2nd order geometric aberrations while imposing only mild constraints on the cell components. ii) The vertical and horizontal chromatic aberrations are cancelled by two sextupoles per unit cell.

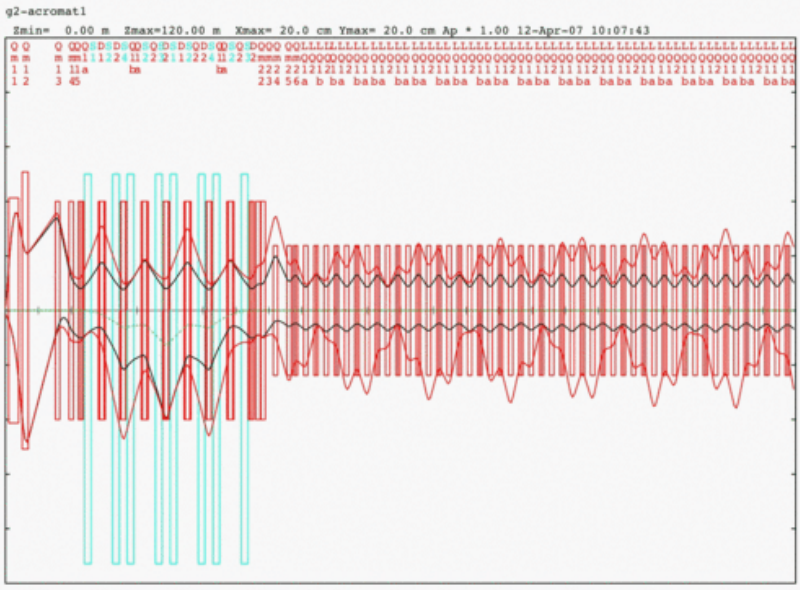



Figure 8: Transport envelopes for achromat for 1st and 2nd order, 3% $\delta p$.

In a first step a schematic design was developed which consists of simple separated function magnets and does not respect the physical boundaries of the E821 beam line. The transport envelopes in fig. 8 show that in spite of the perfect achromat in the bending section, the FODO matching is not perfect. This behavior is traced to second order chromatic effects from the pion injection magnets. Apparently a compromise between 2nd order correction over the whole pion injection beam line must be optimized.

### 3.2.2 Performance

Two decay Turtle simulations compare an achromat design and the current beamline design. The input file places the achromat pion injection in front of a lattice with the same length as the backward decay lattice in previous simulations. The results are summarized in Fig. 9.

In each summary plot panel the momentum distribution of pions which transport in the lattice, and thus produce muons, and the muon momentum near the magic momentum ($\pm$ 0.25%) and finally the xy distribution of the muons in the above momentum range and their polarization distribution is displayed.

The relative figures of merit are related in the plot shown in Fig. 10. Here we see that the achromat compares favorably with the previous backward decay scenario in the current pion injection, and even begins to appear similar to the best forward decay case.

The achromat as employed in these transport and turtle files is designed for a 3% momentum bite. This version does have the same total bending angle as the current beam line. It is possible to transport up to 5% momentum width in a system which is shorter by using higher quadrupole fields and quad-sextupole multipole magnets instead of separate function magnets, which reduces the length considerably(15.3 m compared to 26.02 m here).

These results, though promising, are very preliminary and need to be checked. In particular, the FOM improvement is surprising, given the marginal improvement in the transport envelopes. In addition, the final phase space acceptance factors p($\epsilon$) have to be carefully included. As demonstrated above, they drastically affect the FOM of the backward concept.

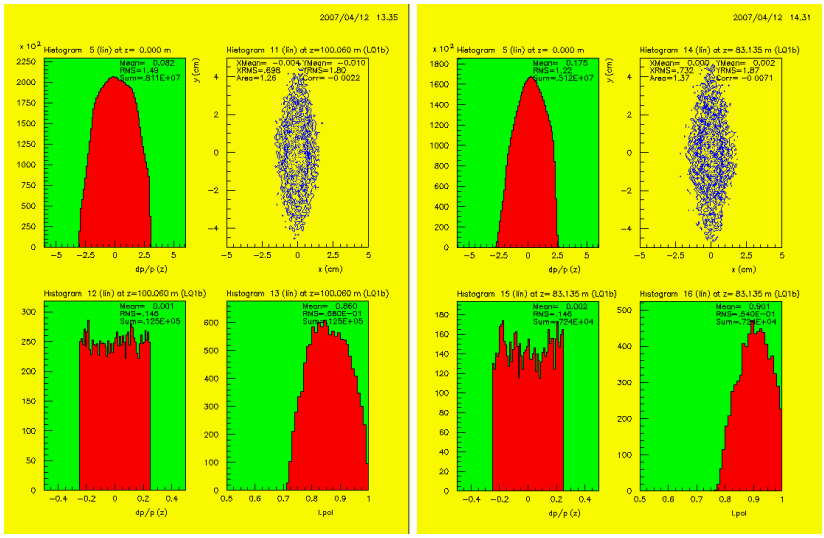


Figure 9: Turtle for achromat (left) and standard backward beam (right)

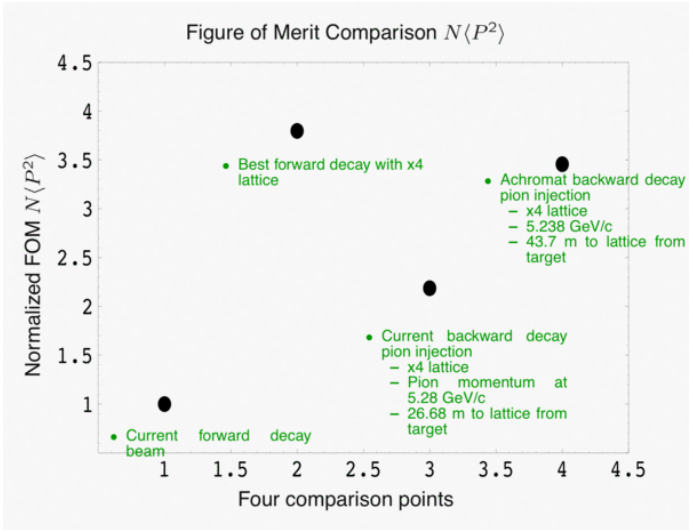



Figure 10: FOM comparison, not including inflector-ring acceptance factor

### 3.2.3 Realistic Design

Once the virtue of the achromat is established, a realistic design with combined function magnets must try to fit the channel to the physical E821 beamline boundaries.

## 3.3 Muon Accumulator Ring

The MAR performance is studied by the following straw man design (Fig. 4). The beam is deflected from the FODO into a 180° return arc. The arc is followed by a 11 quad FODO with identical elements as the original FODO. Another 180° arc connects this beam line to the original FODO. Thus the original FODO provides one of the legs of the racetrack ring. The bends are designed as two 2nd order achromats with 3% momentum acceptance. We choose a 2x FODO for this design, to employ a similar number of quads as in the forward 4X concept. Initially the beam is steered into the MAR. After sufficient turns the beam is kicked towards the transfer line to the g-2 ring with a fast kicker. This operation mode prevents AGS background to enter the ring and also can eliminate protons from the fast extraction pulse. The pions need ≈450 ns per turn, the protons lag behind by ≈xx ns per turn.

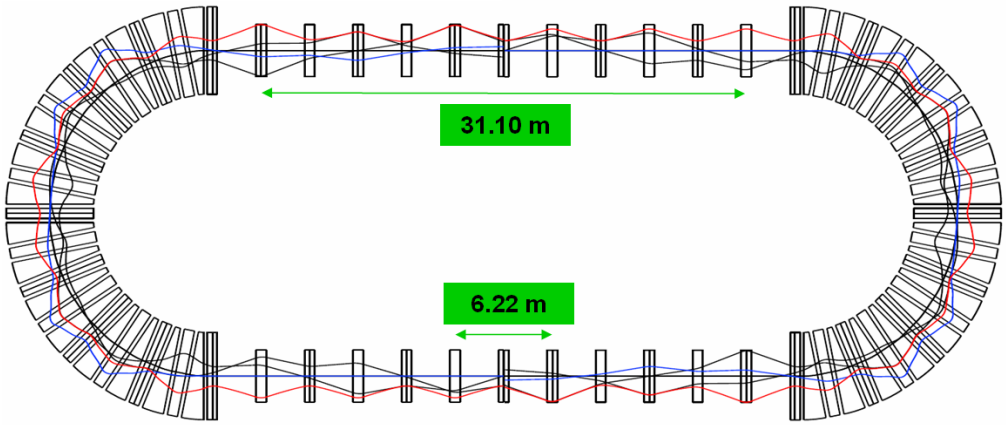



Figure 11: Sketch of MAR from Cosy-Infinity

| | Ave. Pol. | Pol. Width | norm # mu | xx' | yy' | p12 | p34 | NP2 | FOM (Norm.) |
|---|---|---|---|---|---|---|---|---|---|
| E821 | 0.951 | 0.013 | 1.00 | 3.07 | 3.61 | 0.52 | 0.59 | 1.00 | 1.00 |
| MAR 0 turn | 0.972 | 0.0141 | 4.89 | 2.86 | 4.49 | 0.54 | 0.51 | 4.62 | 4.20 |
| MAR 4 turn | 0.972 | 0.0187 | 13.3 | 3.00 | 3.72 | 0.53 | 0.58 | 12.59 | 12.55 |
| MAR 8 turn | 0.972 | 0.0182 | 11.06 | 2.45 | 3.75 | 0.60 | 0.57 | 10.45 | 11.81 |

Table 6: TURTLE simulation of FOM and E821, for comparison. $p_\pi/p_\mu = 1.005$. Sara1.xls

The design of a kicker for the magic muon phase space and its physical implementation needs to be studied. The presented design is for demonstration only and the massive arcs are clearly over designed. One achromat with larger openings at the maximum dispersion should be sufficient. The concept needs to be optimized regarding properties, kickability and cost. Fig. 11 sketches the ring and Fig 12 shown the TRANSPORT envelopes for a few turns.

The simulated performance of the ring is shown in Fig. 13 and Table 3.3. The numbers at 0 turns refer to the FODO location, just before the beam is bent into the first arc. The flux values indicate an optimum at 4 turns, however, with some loss of flux 8 turns would result in more than 10 fold higher pion suppression. The FOM is calculated relative to the standard E821 forward beam. Already at 0 turns, the combined effects of ideal and wide $p_\pi$ selection and the 2x FODO lead to a FOM increase by 4.2. The muon collection in MAR eventually leads to a gain factor 12.5. The gain factor should be optimized as function of $p_\pi/p_\mu$. The results quoted in Table 3.3 are for $p_\pi/p_\mu = 1.005$, the final comparison is performed with a $p_\pi/p_\mu = 1.01$ simulation, which resulted in a slightly higher gain factor of 13.8. Quadrupling the FODO magnets would increase the intensity further, if that is required.

Overall, this is a exciting improvement which would enhance the reach of the experiment dramatically.

The g-2 rotation in the arcs will lead to some loss of polarization. The muon flux in MAR (Fig. 13) can be fitted as $N_\mu(n) \approx 1 - 0.7\, e^{-1.18n}$, where n is the number of turns in MAR. We assume that both arc and FODO have the length of 1/4 turn. After each arc the phase of the muons will have changed by $\phi_{arc} = a_\mu \gamma \pi = 6.2°$. Defining the initial polarization at turn 0 equal 1, the muons produced after n arcs will have a relative polarization $cos(n\phi_{arc})$. The fractional contributions to the different polarizations is shown in fig. 14. The average polarization remains high at 0.986.

The debunching of the fast pulse due to path length difference should also be determined by simulation. It would affect the pulse width and thus the matching to an optimized fast kicker in the g-2 ring. Finally, we should carefully study the phase space to polarization correlation. But no show stoppers are expected by these effects.

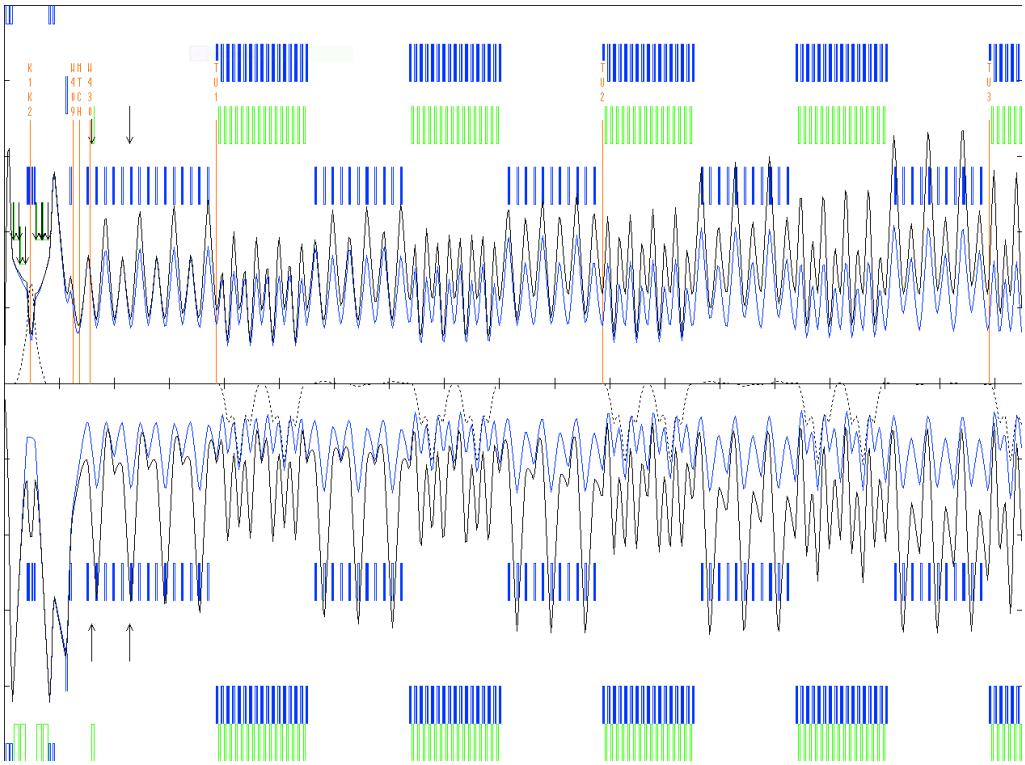


Figure 12: Transport envelopes for the first few MAR turns. $\delta_\pi$=1.5% and 3%, respectively.

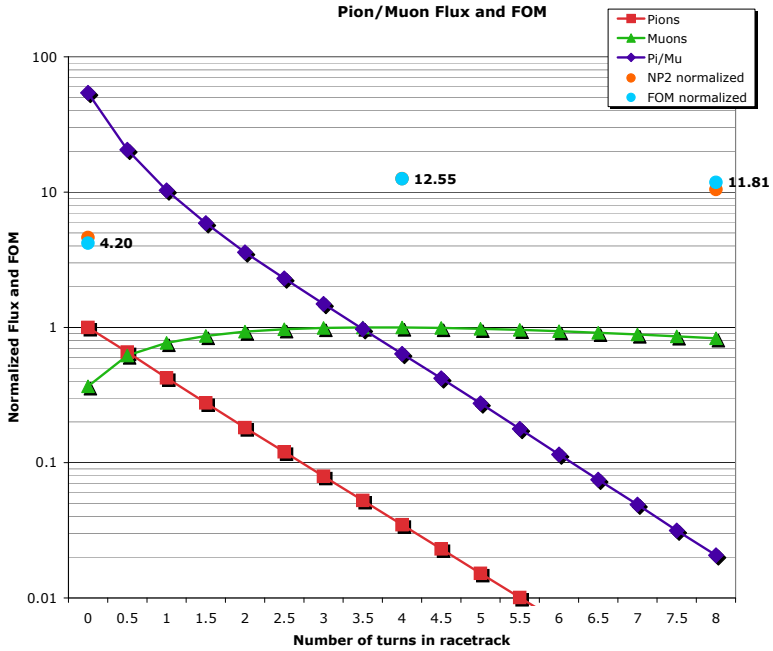



Figure 13: Turtle predications for MAR. Fluxes normalized to largest point, FOM normalized to E821 standard conditions.

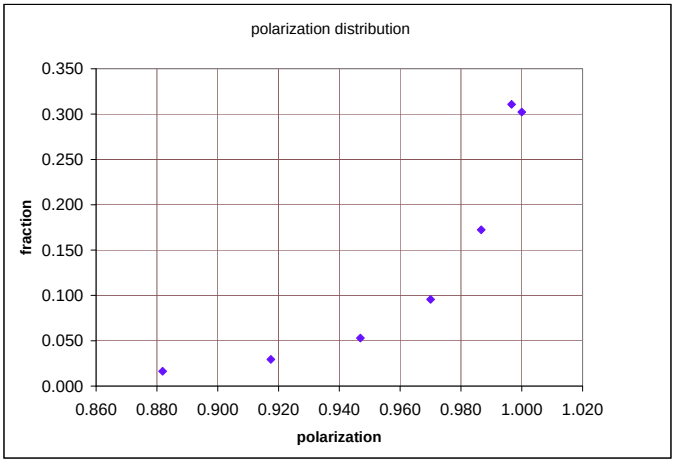



Figure 14: Simple estimate of different relative polarizations accumulated in MAR

## 3.4 Closed/Open Inflector

The present E821 inflector has closed ends, i.e. the muons have to transverse the superconducting coils when entering and leaving the 1700 mm long inflector. The E969 proposal estimates that replacing the present inflector with an open ended one increases the collected muon flux in the g-2 ring by a factor of two. From the studies above it is clear that acceptance matching to the inflector ring critically contributes to the FOM of different tunes. Thus the multiple scattering contributions from the inflector with/without window are expected to differently impact the FOM of the different beam transports systems considered. All these optimization issues are tightly interlinked and thus cannot be considered independently. While a detailed Monte Carlo study of the inflector is needed, we sketch out a first estimate below.

| | | | | | | | | | | | window | no wind | no wind/E821 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Condition | xx' | yy' | x'@x=0.9 | y'@y=2.8 | X' | Y' | xX' | yY' | acc_x | acc_y | norm FOM | norm FOM | norm FOM |
| for 1x | 3.18 | 4.22 | 3.53 | 1.51 | 4.53 | 3.20 | 4.07 | 8.97 | 0.35 | 0.50 | 1.00 | 1.00 | 1.61 |
| for 2x | 3.41 | 4.51 | 3.79 | 1.61 | 4.73 | 3.25 | 4.26 | 9.11 | 0.36 | 0.51 | 2.61 | 2.27 | 3.65 |
| for 4x | 3.26 | 5.48 | 3.62 | 1.96 | 4.60 | 3.44 | 4.14 | 9.63 | 0.35 | 0.53 | 3.93 | 3.03 | 4.86 |
| for 4x pi+3% | 3.82 | 6.01 | 4.24 | 2.15 | 5.10 | 3.55 | 4.59 | 9.94 | 0.38 | 0.54 | 3.85 | 2.23 | 3.59 |
| for 4x pi+5% | 4.74 | 6.54 | 5.27 | 2.34 | 5.98 | 3.67 | 5.38 | 10.27 | 0.43 | 0.55 | 3.60 | 1.45 | 2.32 |
| back 1x | 2.44 | 2.81 | 2.71 | 1.00 | 3.92 | 3.00 | 3.53 | 8.40 | 0.31 | 0.48 | 0.57 | 1.01 | 1.62 |
| back 2x | 3.62 | 4.65 | 4.02 | 1.66 | 4.92 | 3.28 | 4.43 | 9.18 | 0.37 | 0.51 | 1.95 | 1.53 | 2.47 |
| back 4x | 4.03 | 6.16 | 4.48 | 2.20 | 5.30 | 3.58 | 4.77 | 10.03 | 0.39 | 0.54 | 3.00 | 1.59 | 2.55 |
| back 4x qs | 3.98 | 6.85 | 4.42 | 2.45 | 5.25 | 3.74 | 4.73 | 10.47 | 0.39 | 0.56 | 3.36 | 1.62 | 2.60 |

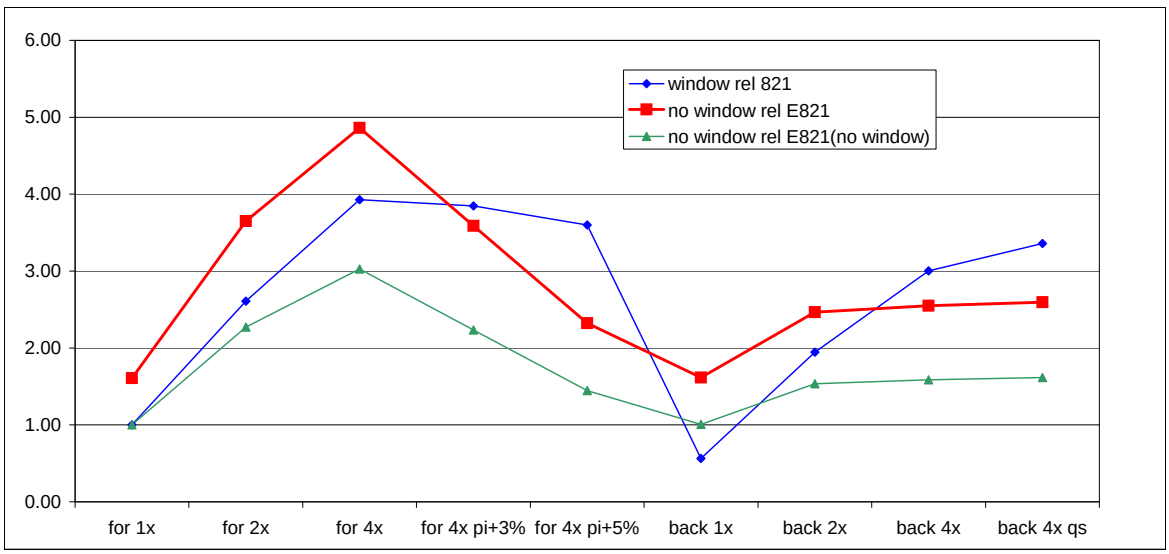



Figure 15: Forward-backward normalized FOM including open/closed inflector estimates

Upon inflector entrance the muons pass through a 1mm Al window (1.1% $X_0$), 3 mm Al ? (3.3% $X_0$) and two layers of superconducting coil (11.2 mm Al, 12% $X_0$, 0.8 mm Cu 5.6 % $X_0$ and 0.8 mm Nb/Ti 4.6% $X_0$). Naively adding these fractions, results in a total thickness of 27% of a radiation length $X_0$ which causes a planar rms scattering angle $\theta_0$ of $\approx$ 2.1 mrad. The naive model for including the scattering effect in the FOM is as follows. At inflector entrance (or center) we assume that the beam optics produced a waist with rms x=9 mm and rms y=28 mm. The corresponding rms angles x' and y' are calculated from the beam emittance. Due to the multiple scattering these rms angles are widened to $X' = \sqrt{(x'^2 + 2 \times \theta_0^2)}$. This leads to a widened beam emittance of $\epsilon_x = \pi x X'$. The same procedure is applied to the y plane. The resulting normalized FOM is displayed in figure 15. Three cases can be distinguished. The present E821 beam with the closed inflector is compared to beam variants with closed inflector (curve: window relative to E821). The E821 beam with a open inflector can be compared to beam variants with open inflector (curve: no window relative to E812 (no window)). Finally, the present E812 tune with closed inflector can be compared to the FOM of beam variants with open inflector ( curve: no window relative to E821). This is the most relevant version for evaluating the future beam options. The relative FOM behaviour of the variants differ significantly, which can be understood by the fact that scattering is more important if the undisturbed emittance is smaller. The combined FOM gain of opening the inflector and selecting forward 4x +3% is 3.59 and of selecting backward 4x is 2.55, both falling short of the expected 5 fold beam increase.

# 4 Othere Issues

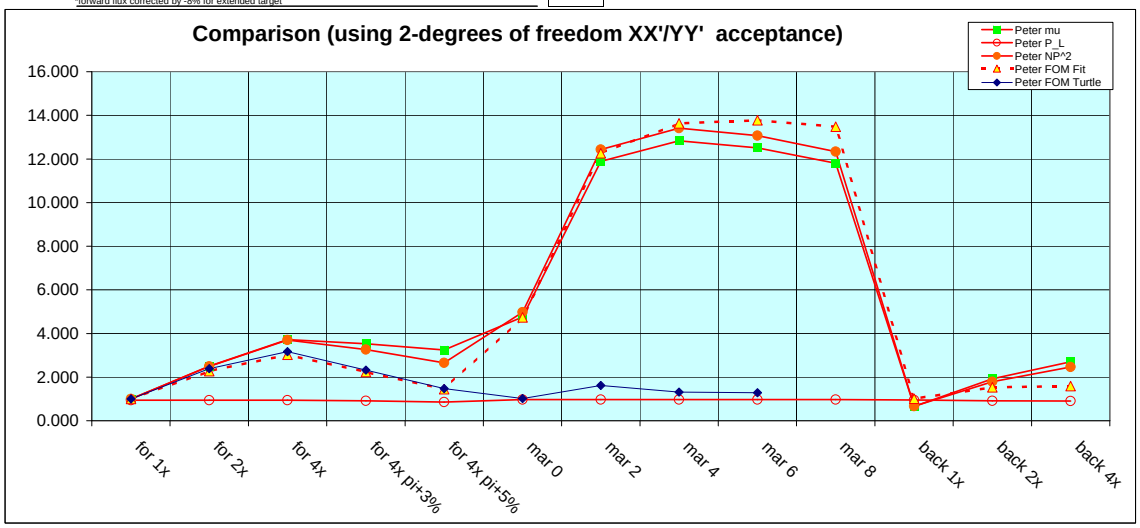
# 5 Cost Comparison

# 6 Overall Comparison

Fig. 16 show the overall comparison. Although the presented studies are preliminary, the muon accumulator ring promises a dramatic improvement in the g-2 experimental quality compared to the options previously considered. The combination of MAR and opening the inflector end should boost the magic muon intensity by a factor of ≈20, while eliminating the hadronic flash, fast extraction protons and other AGS background. The division of the accelerator intensity in more bunches (e.g. from 12 to 36 bunches in the AGS) is essential, to fully utilize this potential by keeping the instantaneous rates low. The "reserve" intensity will also allow more agressive RF and scraping methods after muon injection into the g-2 ring, to improve the stored muon quality regarding coherent betatron motion. Pending future advancement based on cooled muon beams etc., the proposed scheme concludes the logical sequence of seminal ideas from pion injection, to muon injection and finally to muon accumulation before injection, which were instrumental for the spectacular improvements in the precision of the muon anomaly over the last 40 years.

| Condition | N | X | Y | P | sig | Ncorr* | Norm N | ABS(P) | NP2 | Norm NP2 | within XX' | within YY' | FOM | Norm FOM | XX' | YY' | within XX' | within YY' | FOM | Norm FOM | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| for 1x | 3406 | 3.18 | 4.22 | -0.945 | 0.014 | 3134 | 1.000 | 0.945 | 0.893 | 1.000 | 0.87 | 0.87 | 0.756 | 1 | 1.51 | 1.52 | 0.53 | 0.53 | 0.28 | 1 | for1kg |
| for 2x | 8524 | 3.41 | 4.51 | -0.943 | 0.014 | 7842 | 2.503 | 0.943 | 2.226 | 2.492 | 0.84 | 0.84 | 1.769 | 2.338 | 1.41 | 1.42 | 0.51 | 0.51 | 0.64 | 2.27 | for2kg |
| for 4x | 12674 | 3.26 | 5.48 | -0.942 | 0.015 | 11660 | 3.721 | 0.942 | 3.303 | 3.698 | 0.86 | 0.76 | 2.405 | 3.180 | 1.47 | 1.17 | 0.52 | 0.44 | 0.85 | 3.03 | for4kg |
| for 4x pi+3% | 12019 | 3.82 | 6.01 | -0.909 | 0.015 | 11057 | 3.529 | 0.909 | 2.917 | 3.265 | 0.79 | 0.71 | 1.842 | 2.435 | 1.26 | 1.06 | 0.47 | 0.41 | 0.63 | 2.23 | for4kp12g |
| for 4x pi+5% | 11049 | 4.74 | 6.54 | -0.854 | 0.015 | 10165 | 3.244 | 0.854 | 2.367 | 2.650 | 0.69 | 0.67 | 1.227 | 1.622 | 1.01 | 0.98 | 0.40 | 0.39 | 0.41 | 1.45 | for4kp32g |
| mar 0 | 16200 | 3.19 | 4.52 | -0.966 | 0.024 | 14904 | 4.756 | 0.966 | 4.441 | 4.972 | 0.87 | 0.84 | 3.637 | 4.809 | 1.50 | 1.42 | 0.53 | 0.51 | 1.33 | 4.74 | marh |
| mar 2 | 40500 | 3.5 | 3.9 | -0.966 | 0.022 | 37260 | 11.891 | 0.966 | 11.102 | 12.429 | 0.83 | 0.90 | 9.274 | 12.260 | 1.37 | 1.64 | 0.50 | 0.56 | 3.45 | 12.26 | marh |
| mar 4 | 43700 | 3.46 | 3.77 | -0.966 | 0.022 | 40204 | 12.830 | 0.966 | 11.979 | 13.411 | 0.83 | 0.91 | 10.191 | 13.472 | 1.39 | 1.70 | 0.50 | 0.57 | 3.84 | 13.63 | marh |
| mar 6 | 42600 | 3.34 | 3.7 | -0.966 | 0.022 | 39192 | 12.507 | 0.966 | 11.677 | 13.073 | 0.85 | 0.92 | 10.174 | 13.451 | 1.44 | 1.73 | 0.51 | 0.58 | 3.88 | 13.77 | marh |
| mar 8 | 40200 | 3.19 | 3.67 | -0.966 | 0.022 | 36984 | 11.803 | 0.966 | 11.019 | 12.337 | 0.87 | 0.92 | 9.834 | 13.001 | 1.50 | 1.74 | 0.53 | 0.58 | 3.80 | 13.48 | marh |
| back 1x | 2070 | 2.44 | 2.81 | 0.948 | 0.034 | 2070 | 0.661 | 0.948 | 0.594 | 0.666 | 0.95 | 0.98 | 0.618 | 0.818 | 1.97 | 2.28 | 0.63 | 0.68 | 0.28 | 1.01 | back1g |
| back 2x | 6028 | 3.62 | 4.65 | 0.911 | 0.049 | 6028 | 1.924 | 0.911 | 1.601 | 1.793 | 0.82 | 0.83 | 1.215 | 1.606 | 1.33 | 1.39 | 0.48 | 0.50 | 0.43 | 1.53 | back2m10g |
| back 4x | 8470 | 4.03 | 6.16 | 0.9 | 0.053 | 8470 | 2.703 | 0.900 | 2.197 | 2.460 | 0.77 | 0.70 | 1.321 | 1.747 | 1.19 | 1.04 | 0.45 | 0.41 | 0.45 | 1.59 | back4m13g |

*forward flux corrected by -8% for extended target




| | | | | | | | | | | | | | | window | no wind | no wind/E821 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Condition | N | xx' | yy' | x'@x=0.9 | y'@y=2.8 | X' | Y' | xX' | yY' | acc_x | acc_y | Norm NP2 | FOM | norm FOM | norm FOM | norm FOM |
| for 1x | 3406 | 3.18 | 4.22 | 3.53 | 1.51 | 4.53 | 3.20 | 4.07 | 8.97 | 0.35 | 0.50 | 1.00 | 0.17 | 1.00 | 1.00 | 1.61 |
| for 2x | 8524 | 3.41 | 4.51 | 3.79 | 1.61 | 4.73 | 3.25 | 4.26 | 9.11 | 0.36 | 0.51 | 2.49 | 0.45 | 2.61 | 2.27 | 3.65 |
| for 4x | 12674 | 3.26 | 5.48 | 3.62 | 1.96 | 4.60 | 3.44 | 4.14 | 9.63 | 0.35 | 0.53 | 3.70 | 0.68 | 3.93 | 3.03 | 4.86 |
| for 4x pi+3% | 12019 | 3.82 | 6.01 | 4.24 | 2.15 | 5.10 | 3.55 | 4.59 | 9.94 | 0.38 | 0.54 | 3.27 | 0.67 | 3.85 | 2.23 | 3.59 |
| for 4x pi+5% | 11049 | 4.74 | 6.54 | 5.27 | 2.34 | 5.98 | 3.67 | 5.38 | 10.27 | 0.43 | 0.55 | 2.65 | 0.63 | 3.60 | 1.45 | 2.32 |
| mar 0 | 16200 | 3.19 | 4.52 | 3.54 | 1.61 | 4.53 | 3.26 | 4.08 | 9.12 | 0.35 | 0.51 | 4.97 | 0.88 | 5.03 | 4.74 | 7.61 |
| mar 2 | 40500 | 3.5 | 3.9 | 3.89 | 1.39 | 4.81 | 3.15 | 4.33 | 8.83 | 0.36 | 0.50 | 12.43 | 2.25 | 12.90 | 12.26 | 19.70 |
| mar 4 | 43700 | 3.46 | 3.77 | 3.84 | 1.35 | 4.77 | 3.13 | 4.30 | 8.77 | 0.36 | 0.50 | 13.41 | 2.40 | 13.77 | 13.63 | 21.90 |
| mar 6 | 42600 | 3.34 | 3.7 | 3.71 | 1.32 | 4.67 | 3.12 | 4.20 | 8.74 | 0.35 | 0.49 | 13.07 | 2.29 | 13.15 | 13.77 | 22.13 |
| mar 8 | 40200 | 3.19 | 3.67 | 3.54 | 1.31 | 4.53 | 3.12 | 4.08 | 8.73 | 0.35 | 0.49 | 12.34 | 2.11 | 12.12 | 13.48 | 21.65 |
| back 1x | 2070 | 2.44 | 2.81 | 2.71 | 1.00 | 3.92 | 3.00 | 3.53 | 8.40 | 0.31 | 0.48 | 0.67 | 0.10 | 0.57 | 1.01 | 1.62 |
| back 2x | 6028 | 3.62 | 4.65 | 4.02 | 1.66 | 4.92 | 3.28 | 4.43 | 9.18 | 0.37 | 0.51 | 1.79 | 0.34 | 1.95 | 1.53 | 2.47 |
| back 4x | 8470 | 4.03 | 6.16 | 4.48 | 2.20 | 5.30 | 3.58 | 4.77 | 10.03 | 0.39 | 0.54 | 2.46 | 0.52 | 3.00 | 1.59 | 2.55 |

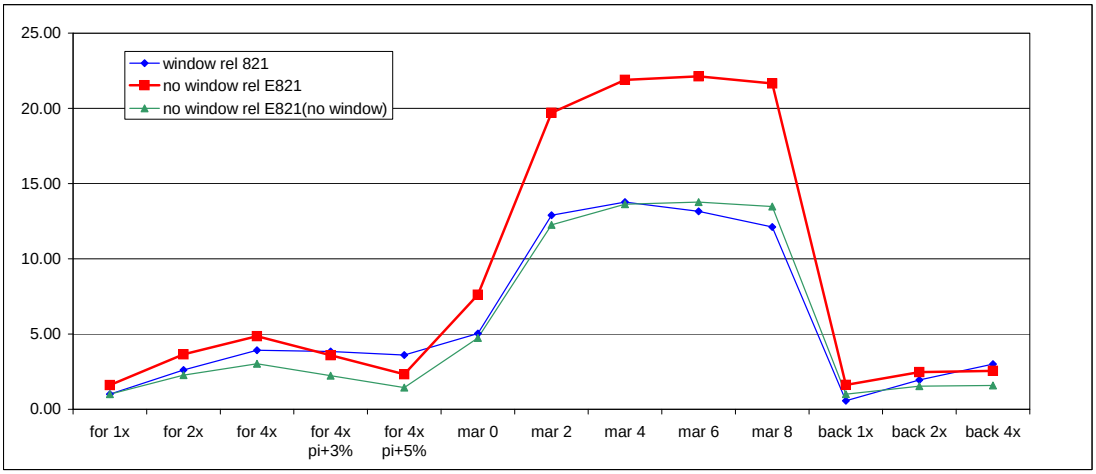



Figure 16: Top: Overal simulation results. Bottom: Overal simulation results including inflector end effect. MAR $p_\pi/p_\mu = 1.01$. Results07.xls

# References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# 7 Appendix

This paper has the associated webpage

`http://www.npl.uiuc.edu/twiki/bin/view/Main/PeterKg2Beam`

where supporting information can be stored.

## 7.1 Beam Hardware

## 7.2 Transport Theory

### 7.2.1 FODO Matching Strategies

Paul and Sara will include the matching conditions and the practical strategy used.

There are several methods one might use to calculate or find the initial sigma matrix conditions a beam should have in order to match a lattice system. One may use the beta function (beam elvelope) and matrix formalism to calculate the optimal initial conditions, for example. Another method is to vary the initial sigma matrix elements such that the beam size is the same after one unit cell of a lattice. The result of this fit are the conditions that the beam sigma matrix must meet.

Once the desired beam envelope ($\sigma$ matrix) is established a match can be made with a simple fit. If the fit is done to the center of the first quadrupole magnet the condition will be that $\sigma_{12} = \sigma_{21} = \sigma_{34} = \sigma_{43} = 0$. Two ratios, $r_1 = \frac{\sigma_{11}}{\sigma_{22}}$ and $r_2 = \frac{\sigma_{33}}{\sigma_{44}}$, are defined by the remaining elements for the $xx'yy'$ phase space. The ratios $r_1$ and $r_2$ will have to be equal to the ratios $r_x$ and $r_y$ respectively, set by the characteristics of the lattice (spaceing, quad. strength, particle momentum, etc.) for a beam to be well transported. The fit for the beam into the lattice can be established with four conditions

- $\sigma_{12} = 0$
- $\sigma_{34} = 0$
- $r_1 = r_x$
- and $r_2 = r_y$.

Such a fit is easily implemented in a program like COSY INFINITY, where one can flexibly define fit functions as functions of the sigma matrix elements. It can also be done in Transport with the 24 card options available. In general I have little experience with this in Transport, where it is more standard to us the matrix elements themselves and not the ratios as I have described above.

### 7.2.2 FODO Transport

#### Number of Quads

Muons from pion decay are emitted along the decay channel isotropically in the pion rest frame. The laboratory momentum and angle are governed by two-body decay kinematics and the initial pion momentum and angle. Only muons in a narrow momentum band near the magic momentum of 3.094 GeV/c are stored by the ring. In addition, the muons must remain within the admittance of the channel and storage ring. For a given emittance, the maximum extent of the beam envelope xmax is proportional to the square root of the maximum of the beta function of the channel, $\beta_{max}$. For a FODO lattice, $\beta_{max}$ is proportional to the length of the FODO cell having a fixed ratio of focal length f to cell length L. The ratio f/L determines the phase advance per cell, $\mu$. Thus if the length of the cell were reduced by a factor of two and is not changed, $\beta_{max}$ is also reduced by a factor of two, and $x_{max}$ is reduced by a factor of $\sqrt{2}$. Since this reduction occurs in both transverse planes, the increase in transmitted flux could be as large as a factor of two.

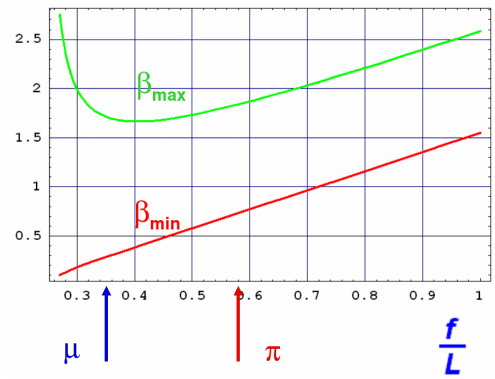



Figure 17: Beta function for pions and muons in the FODO.

**Momentum acceptance of FODO**

The FODO can transport 3.1 and 5.2 GeV/c particles without much losses for the correct choice of lattice parameters. Fig 17 shows that the FODO tuned for magic muons will also transport the 5.2 GeV/c pions needed for backward decay. Paul, do you want to add explanations to this?.

### 7.2.3 K. Brown's Achromat

A second order achromat is a transport system for which all transverse chromatic terms of the transfer matrices up to second order vanish. A useful special case which maps the the input transverse phase space to an identical image at the output can be described with the condition R=1 and T=0, i.e. the matrix elements of the first order transfer matrix $R_{ij} = \delta_{ij}$ and $T_{ijk} = 0$ for all i,j,k. The solution is a series of four identical unit cells much like the quadrupole lattice with a systematic placement of dipole and sextupole fields amongst the quadrupole magnets [5, 7].

K. Brown demonstrated [5] that all second-order geometric contributions can be canceled in a system that is periodic with four or more unit cells. The only requirement is that the system have $n$ unit cells and $R_{total}$ be the identity matrix in the $xx'yy'\delta$ coordinate space. The unit cells can include dipole, quadrupole and sextupole magnets, and multipoles there of. The chromatic contributions do not necessarily cancel, but, since there is no constraint on the sextupole fields, they can be used exclusively to cancel chromatic aberrations. This cancellation is proved below. This knowledge has been employed in other beamlines that transport wide momentum bands of particles and/or minimize second-order effects. .

For any periodic system having $n$ unit cells, take $R(i)$ ($T(i)$) to be the first (second) order transformation after $i$ unit cells. The total first-order matrix of a system of $n$ identical unit cells is calculated knowing $R_{total} = R(n) = R(n-1)R(1) = R(n-1)R(1)$, where $R(1) = R_{cell}$, and $R(n-1) = (R_{cell})^{n-1}$. The total second-order matrix for a system with $n$ unit cells is [7] expressed as

$$T(n)_{ijk} = \sum_l R(n-1)_{il}T(1)_{ljk} + \sum_{lm} T(n-1)_{ilm}R(1)_{lj}R(1)_{mk} \text{ and} \tag{3}$$

$$T(n) = R(1)T(n-1) + T(1)R(n-1)R(n-1), \tag{4}$$

where the matrix multiplication is identical in both equations. These two relations are succinctly explained that, if the achromat is split into two parts, it is equivalent to consider the first unit cell and then $n-1$ cells together, or $n-1$ unit cells and the last. The conclusion is that the elements of $T_{total}$ (the second-order matrix of the entire system) can be expressed as linear combinations of the elements of $R_{cell}$ and the $T_{total}$ elements themselves in the relation

$$T(n)_{ijk} = \sum_{nlm} R(1)_{in}T(n)_{nlm}R(1)^{-1}_{lj}R(1)^{-1}_{mk}. \tag{5}$$

Now assume $R_{cell}$ is like

$$R_{cell} = \begin{bmatrix} 0 & R_{12} & R_{16} \\ R_{21} & 0 & R_{26} \\ 0 & 0 & 1 \end{bmatrix} \rightarrow (R_{cell})^4 = \begin{bmatrix} R_{12}^2 R_{21}^2 & 0 & (1+R_{12}R_{21})(R_{16}+R_{12}R_{26}) \\ 0 & R_{12}^2 R_{21}^2 & (1+R_{12}R_{21})(R_{16}R_{21}+R_{26}) \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ because } 1 + \mathrm{R}_{12}\mathrm{R}_{21} = 0 \tag{7}$$

in the $xx'\delta$ coordinate space, the bend plane. Each unit cell is chromatic ($R_{16} \neq R_{26} \neq 0$), yet the entire system of four unit cells is achromatic, and the first-order transformation is the identity matrix (if $R_{11} = R_{22} = 0$ and $1 + R_{12}R_{21} = 0$). The total first-order transformation $R_{total}$ is the identity matrix in the $xx'yy'\delta$ coordinates if $R_{33} = R_{44} = 0$ and $1 + R_{34}R_{43} = 0$. With this form of $R_{cell}$ and Equation 5, it is possible to show that the geometric second-order matrix elements are zero.

The fact that each unit cell is chromatic in first-order is a very loose constraint on the dipole properties of the achromat. A strong first-order constraint is implied if $R_{cell\ ii} = 0$ for $i = 1, 2, 3,$ and 4. This demands point-to-point and parallel-to-parallel focusing in each unit cell, in both planes. A quadrupole lattice is the canonical example meeting such a condition. The achromat unit cell begins at the center of a focusing quadrupole, continues through a dipole, a defocusing quadrupole, a second identical dipole and ends at the center of a quadrupole identical to the first. To meet the specific conditions $1 + R_{12}R_{21} = 0$ and $1 + R_{34}R_{43} = 0$ the dipoles are given a small pole face rotation and the placement between quadrupoles can be varied. This is a bending lattice; the beam will traverse this system much like in a quadrupole lattice but simultaneously be bent. The important difference is that the momentum width of the beam, $\sigma_{66}$, also determines the beam dynamics.

The cancellation of the chromatic second-order matrix elements can now be demonstrated. Equation 5 related the first-order unit cell and total second-order matrix elements. Confine the discussion to the bend plane chromatic elements, $T_{ijk}$ where $i = 1$ or 2 and $k = 6$. The elements $T_{i66}$ are 0 (again from the form of $R_{cell}$ and Equation 5) and $j = 3$ or 4 elements are excluded by mid-plane symmetry. The non-zero contributions are associated with $T_{116}$, $T_{126}$, $T_{216}$, and $T_{226}$. The form of $R_{cell}$ and Equation 5 establish a system of equations [7]

$$T_{116} = -R_{12}R_{21}T_{226} \tag{8}$$
$$T_{226} = -R_{12}R_{21}T_{116} \tag{9}$$
$$T_{126} = -R_{12}R_{12}T_{216} \tag{10}$$
$$T_{216} = -R_{21}R_{21}T_{126}. \tag{11}$$

The last two relations show $R_{12}T_{216} - R_{21}T_{126} = 0$ if $R_{12}R_{21} + 1 = 0$. It follows if a sextupole field degree of freedom is used to cancel $T_{126}$, $T_{216} = 0$ also. The $R_{12}R_{21} + 1 = 0$ condition also reduces the first two expressions to $T_{116} = T_{226}$. If $T_{126} = T_{216} = 0$ the effective total first-order matrix [7] is

$$\det \left| \begin{bmatrix} 1+\delta T_{116} & 0 \\ 0 & 1+\delta T_{226} \end{bmatrix} \right| = 1 + \delta(\mathrm{T}_{116} + \mathrm{T}_{226}) + \mathcal{O}(\delta^2) \tag{12}$$

assuming the momentum width $\delta$ is a linear perturbation. Dropping terms non-linear in $\delta$ and remembering that $\det|R_{total}| = 1$, it follows that $T_{116} = -T_{226}$. The result is $T_{116} = T_{226} = 0$. With one sextupole degree of freedom all second-order bending plane chromatic aberrations can be canceled for the entire system. This is assuming the same sextupole field is applied in each unit cell, just as for the first-order magnets. The non-bend plane chromatic aberrations are quantified with an analogous set of second-order matrix elements ($T_{336}$, $T_{346}$, $T_{436}$, and $T_{446}$) and meet analogous conditions. A second sextupole degree of freedom cancels the non-bend plane matrix elements. Note that this system is chromatic to first- and second-order throughout the series of elements; only at the end is the transport of particles achromatic.

The placement of sextupole fields in the achromat is important because the fields must be applied in such a way that other second- or higher order effects are not made worse. This is because a sextupole magnet acts oppositely in the bend and non-bend plane (studying the matrix elements of a sextupole field [6] demonstrates this). Where there is correction in one plane, aberration is made in

the other. Dispersion ($\sigma_{16}$) and beam size ($\sigma_{11}$ and $\sigma_{33}$) enhance coupling to the sextupole fields to cancel second-order aberrations, which also minimizes the field strengths required.

The optimal sextupole placement for the bend plane is near the center of a quadrupole focusing in the bend plane, where the beam size ($\sigma_{11}$) is maximum relative to the non-bend plane beam size ($\sigma_{33}$). Conversely the sextupole placement for the non-bend plane should be where the beam is largest in the non-bend plane and minimal in the bend plane, near the center of a quadrupole defocusing in the bend plane. This observation fits well with the quadrupoles of alternating polarity and the first order beam dynamics. The sextupole field associated with the bend plane is typically twofold larger than the non-bend plane field, and opposite in sign. The sextupole fields are also small compared to the first-order fields ($\approx$ 1.6 and 0.8 kG compared to 15 and 24 kG for the quadrupoles and dipoles in the example presented here). The substantial bend plane correction introduces as little aberration as possible to the non-bend plane beam dynamics. These results indicate that the sextupoles are well placed.

## 7.3 Transport and Turtle Files

For each tune the following information is provided. Transport envelopes for $\delta p$=0.5 and 2 %, respectively, using the following input card:

```
1.0  0.2616  17   0.188  65   8.776   2.  3.1523/5.280  /pion/;
```

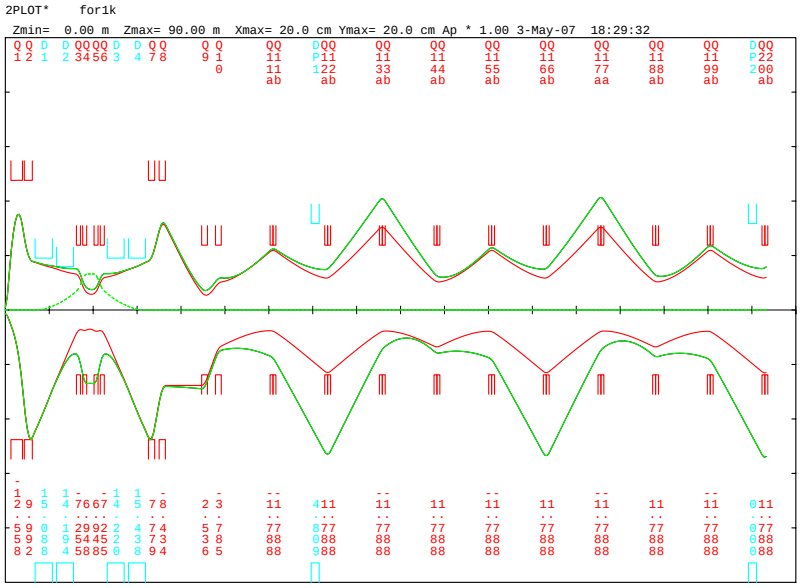
pion (top row) and muon properties at end of FODO; TURTLE input file.

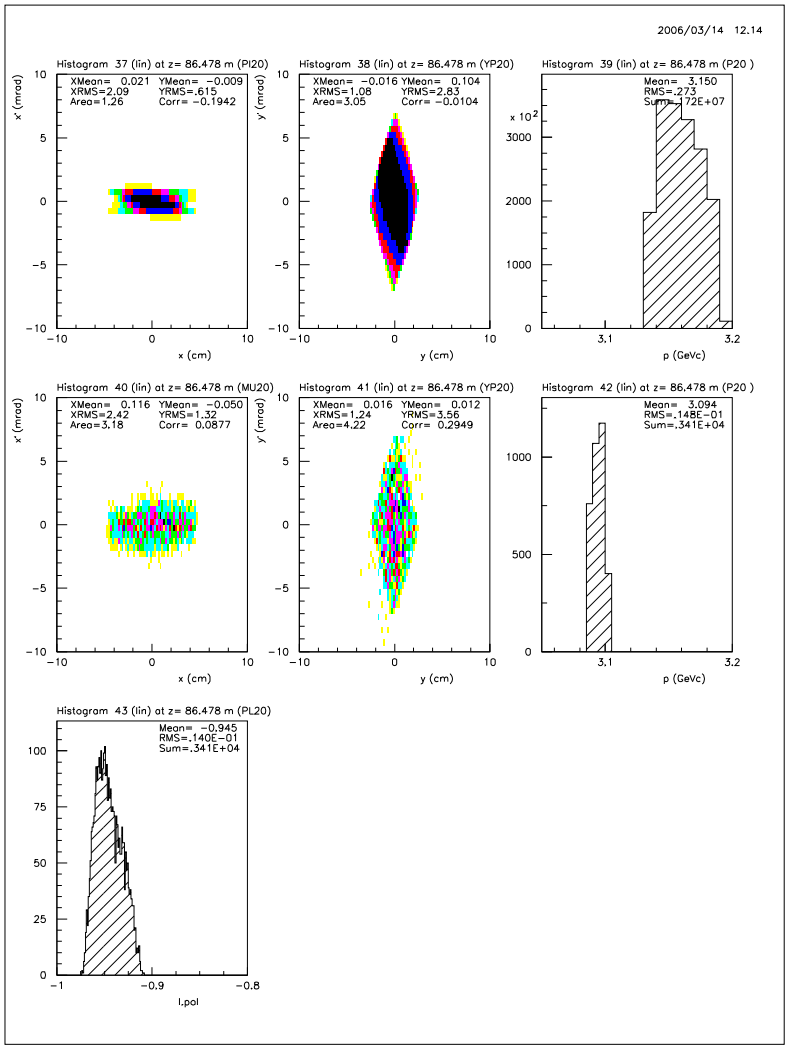
### 7.3.1 Forward 1X

```
/*PLOT*    for1k/
          50000000
( injection scale        1.000      )
(fodo scale              1.000      )
      (                    pi      mu  actual              )
      (        nh           0     100     100              )
      (     p min         5.3    3.05    3.05              )
      (     p max         5.4     3.2     3.2              )
      (    t pion   260000.033  26.033  26.033              )
      ( pitch on/off                     1                 )
      (    PL min          -1 PL max     -0.8              )
      (  Aperture                        1                 )
      (   dP muon                     0.25                 )
      ( transport          )
 (magic      3.094      3.0863  3.0940                      )
     ()
     ()
     ()
     ()
    13.0        10                                                    ;
    13.0       110                                                    ;
    16.0       200     /pion/                                         ;
    16.0       201     /muon/                                         ;
    16.0        20          0                                         ;
     1.0    0.2616          50   0.188     100   8.776       3  3.1523 /pion/;
    60.0    0.1396      0.1057       0  26.033                       ;
   -17.0   /ZWEI/                                                     ;
    16.0       190           0      50                               ;
    51.1         1          -1       1     0.1   /3X/                ;
    52.1         2         -25      25       2 /TARG/                ;
    51.1         3          -1       1     0.1   /Y1/                ;
    52.1         4         -75      75       5 /5V05/                ;
    50.1        11        3.05     3.2   0.005   /P1/                ;
     3.0     0.635                                                   ;
      16       100        9.52    9.52                               ;
     5.0      1.32    -12.5580    9.52   /Q1/                        ;
     ()
     3.0     0.203                                                   ;
      16       100        9.52    9.52                               ;
     5.0     0.914      9.9920    9.52   /Q2/                        ;
     3.0     0.314                                                   ;
    20.0       180                                                   ;
    16.0         5        3.81                                       ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.81                                       ;
     6.0       103        3.81                                       ;
     2.0     8.066      /IN1/                                        ;
     4.0    1.9686     15.0880       0   /D1/                        ;
     2.0     8.066     /OUT1/                                        ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.81                                       ;
     6.0       103        3.81                                       ;
    20.0       180                                                   ;
     3.0     0.478                                                   ;
    20.0       180                                                   ;
    16.0         5        3.18                                       ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.18                                       ;
     6.0       103        3.18                                       ;
     2.0    7.4305      /IN2/                                        ;
     4.0    1.9269     14.1940       0   /D2/                        ;
     2.0    7.4305     /OUT2/                                        ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.18                                       ;
     6.0       103        3.18                                       ;
    20.0       180                                                   ;
     3.0     0.335                                                   ;
      16       100        4.76    4.76                               ;
     5.0     0.457    -7.2550    4.76   /Q3/                        ;
     3.0     0.254                                                   ;
      16       100        4.76    4.76                               ;
     5.0     0.457      6.9480    4.76   /Q4/                        ;
     3.0      0.11                                                   ;
     7.0         1           0       0       0       0       0       ;
     6.0         1        2.66       ;       (    0.812 2.06248     )
     6.0       101        2.66                                       ;
     6.0         3        5.08                                       ;
     6.0       103        5.08                                       ;
     3.0     0.108                                                   ;
     6.0         1        2.53       ;       (     1.14  2.8956     )
     6.0       101        2.53                                       ;
     6.0         3        5.08                                       ;
     6.0       103        5.08                                       ;
     7.0        -1           0       0       0       0       0       ;
     3.0       0.2     /K1K2/                                        ;
    51.0         1          -5       5     0.2/X2K12/                ;
    52.0         2         -20      20       2 /K1K2/                ;
    51.0         3          -5       5     0.2/Y2K12/                ;
    52.0         4         -20      20       2/YPK12/                ;
    51.0        11        3.05     3.2    0.01 /PK12/                ;
    52.0         1          -5       5     0.1 /XK12/                ;
     3.0     0.001                                                   ;
     3.0       0.2                                                   ;
    -6.0         6         0.5                                       ;
     7.0      0.53           0       0       0       0       0       ;
     6.0         1        2.53                                       ;
     6.0       101        2.53                                       ;
     6.0         3        5.08                                       ;
     6.0       103        5.08                                       ;
     3.0     0.108                                                   ;
     6.0         1        2.66                                       ;
     6.0       101        2.66                                       ;
     6.0         3        5.08                                       ;
     6.0       103        5.08                                       ;
     7.0     -0.53           0       0       0       0       0       ;
     3.0      0.11                                                   ;
      16       100        4.76    4.76                               ;
     5.0     0.457      6.9480    4.76   /Q5/                        ;
     3.0     0.254                                                   ;
      16       100        4.76    4.76                               ;
     5.0     0.457    -7.2550    4.76   /Q6/                        ;
     3.0    0.3368                                                   ;
    16.0         5        3.81                                       ;
    20.0       180                                                   ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.81                                       ;
     6.0       103        3.81                                       ;
     2.0    7.4305                      /IN3/                        ;
     4.0    1.9231     14.2200       0   /D3/                        ;
     2.0    7.4305                     /OUT3/                        ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.81                                       ;
     6.0       103        3.81                                       ;
    20.0       180                                                   ;
     3.0     0.503                                                   ;
    20.0       180                                                   ;
    16.0         5        3.81                                       ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.81                                       ;
     6.0       103        3.81                                       ;
     2.0     8.066                      /IN4/                        ;
     4.0    1.9241     15.4380       0   /D4/                        ;
     2.0     8.066                     /OUT4/                        ;
     6.0         1       10.16                                       ;
     6.0       101       10.16                                       ;
     6.0         3        3.81                                       ;
     6.0       103        3.81                                       ;
    20.0       180                                                   ;
     3.0     0.337                                                   ;
      16       100        9.52    9.52                               ;
     5.0     0.711      7.7790    9.52   /Q7/                        ;
     3.0     0.508                                                   ;
     5.0     0.711    -8.4340    9.52   /Q8/                        ;
     3.0      4.13                                                   ;
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
      16       100        4.76    4.76                               ;
     5.0      0.66      2.5360    4.76   /Q9/                        ;
     3.0     0.914                                                   ;
     5.0      0.66    -3.7850    4.76  /Q10/                        ;
     3.0     0.432                                                   ;
     3.0    0.0254                     /W409/                        ;
     3.0     2.318                     /MTCH/                        ;
     3.0     0.001                                                   ;
     3.0    2.7814                                                   ;
    53.1                                                             ;
    16.0        11      3.094                                        ;
     6.0       106       0.25        /SQ11a/                         ;
    16.0        11     3.1523                                        ;
     5.0      0.33    -1.7880    4.76 /Q11a/                         ;
    16.0        11      3.094                                        ;
     6.0       106       0.25         /SQ11/                         ;
    16.0        11     3.1523                                        ;
    51.0       101         -5       5     0.2  /X11/                 ;
    52.0       102         -5       5     0.5 /XP11/                 ;
    51.0       103         -5       5     0.2  /Y11/                 ;
    52.0       104         -5       5     0.5 /YP11/                 ;
    51.0       111       3.05     3.2    0.01  /P11/                 ;
    52.0       101         -5       5     0.1  /X11/                 ;
     5.0      0.33    -1.7880    4.76 /Q11b/                         ;
     3.0     0.432                                                   ;
     3.0    0.0254                     /W430/                        ;
     3.0     3.556                                                   ;
    20.0        90                                                   ;
    16.0         5        6.35                                       ;
    -6.0         1        12.7                                       ;
    -6.0       101        12.7                                       ;
    -6.0         3        12.7                                       ;
    -6.0       103        12.7                                       ;
    16.0        11      3.094                                        ;
     6.0       106       0.25         /SP1/                          ;
    16.0        11     3.1523                                        ;
     6.0         1        6.35                                       ;
     6.0       101        6.35                                       ;
     6.0         3        6.35                                       ;
     6.0       103        6.35                                       ;
     2.0    1.1978                     /INP1/                        ;
     4.0    0.9142      4.8092       0  /DP1/                        ;
     2.0    1.1978                     /OUP1/                        ;
    20.0       -90                                                   ;
     3.0     0.635                                                   ;
    16.0        11      3.094                                        ;
     6.0       106       0.25        /SQ12a/                         ;
    16.0        11     3.1523                                        ;
     5.0      0.33    1.7880    4.76 /Q12a/                         ;
    16.0        11      3.094                                        ;
     6.0       106       0.25        /SQ12/                          ;
    16.0        11     3.1523                                        ;
    51.0       101         -5       5     0.2  /X12/                 ;
    52.0       102         -5       5     0.5 /XP12/                 ;
    51.0       103         -5       5     0.2  /Y12/                 ;
    52.0       104         -5       5     0.5 /YP12/                 ;
    50.0       115         -1    -0.8   0.001 /PL12/                 ;
     5.0      0.33    1.7880    4.76 /Q12b/                         ;
     3.0     0.432                                                   ;
```

```
 3.0    0.0254                    /W450/                    ;
 3.0    5.105                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ13a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33    -1.7880    4.76 /Q13a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ13/                      ;
16.0       11     3.1523                                   ;
51.0      101         -5      5     0.2  /X13/             ;
52.0      102         -5      5     0.5 /XP13/             ;
51.0      103         -5      5     0.2  /Y13/             ;
52.0      104         -5      5     0.5 /YP13/             ;
50.0      115         -1   -0.8   0.001 /PL13/             ;
 5.0     0.33    -1.7880    4.76 /Q13b/                    ;
 3.0    0.432                                               ;
 3.0   0.0245                    /W470/                    ;
 3.0    4.343                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SWEEP/                     ;
16.0       11     3.1523                                   ;
 6.0        1       6.35                                   ;
 6.0      101       6.35                                   ;
 6.0        3       6.35                                   ;
 6.0      103       6.35                                   ;
 3.0    0.762                     /SWP/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ14a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33     1.7880    4.76 /Q14a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ14/                      ;
16.0       11     3.1523                                   ;
51.0      101         -5      5     0.2  /X14/             ;
52.0      102         -5      5     0.5 /XP14/             ;
51.0      103         -5      5     0.2  /Y14/             ;
52.0      104         -5      5     0.5 /YP14/             ;
50.0      115         -1   -0.8   0.001 /PL14/             ;
 5.0     0.33     1.7880    4.76 /Q14b/                    ;
 3.0    5.563                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ15a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33    -1.7880    4.76 /Q15a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25        /SQ15/                     ;
16.0       11     3.1523                                   ;
51.0      101         -5      5     0.2  /X15/             ;
52.0      102         -5      5     0.5 /XP15/             ;
51.0      103         -5      5     0.2  /Y15/             ;
52.0      104         -5      5     0.5 /YP15/             ;
50.0      115         -1   -0.8   0.001  /P15/             ;
 5.0     0.33    -1.7880    4.76 /Q15b/                    ;
 3.0    2.782                                               ;
 3.0    2.782                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ16a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33     1.7880    4.76 /Q16a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25        /SQ16/                     ;
16.0       11     3.1523                                   ;
51.0      101         -5      5     0.2  /X16/             ;
52.0      102         -5      5     0.5 /XP16/             ;
51.0      103         -5      5     0.2  /Y16/             ;
52.0      104         -5      5     0.5 /YP16/             ;
50.0      115         -1   -0.8   0.001  /P16/             ;
 5.0     0.33     1.7880    4.76 /Q16b/                    ;
 3.0    5.563                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ17a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33    -1.7880    4.76 /Q17a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25        /SQ17/                     ;
16.0       11     3.1523                                   ;
51.0      101         -5      5     0.2  /X17/             ;
52.0      102         -5      5     0.5 /XP17/             ;
51.0      103         -5      5     0.2  /Y17/             ;
52.0      104         -5      5     0.5 /YP17/             ;
50.0      115         -1   -0.8   0.001  /P17/             ;
 5.0     0.33    -1.7880    4.76 /Q17a/                    ;
 3.0    5.563                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ18a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33     1.7880    4.76 /Q18a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25        /SQ18/                     ;
16.0       11     3.1523                                   ;
51.0      101         -5      5     0.2  /X18/             ;
52.0      102         -5      5     0.5 /XP18/             ;
51.0      103         -5      5     0.2  /Y18/             ;
52.0      104         -5      5     0.5 /YP18/             ;
50.0      115         -1   -0.8   0.001  /P18/             ;
 5.0     0.33     1.7880    4.76 /Q18b/                    ;
 3.0    5.563                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SQ19a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33    -1.7880    4.76 /Q19a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25        /SQ19/                     ;
16.0       11     3.1523                                   ;
51.0        1         -5      5     0.2  /X19/             ;
52.0        2         -5      5     0.5 /PI19/             ;
51.0        3         -5      5     0.2  /Y19/             ;
52.0        4         -5      5     0.5 /YP19/             ;
50.0       11       3.05    3.2   0.005  /P19/             ;
51.0      101         -5      5     0.2  /X19/             ;
52.0      102         -5      5     0.5 /MU19/             ;
51.0      103         -5      5     0.2  /Y19/             ;
52.0      104         -5      5     0.5 /YP19/             ;
50.0      115         -1   -0.8   0.001 /PL19/             ;
 5.0     0.33    -1.7880    4.76 /Q19b/                    ;
 3.0    4.013                                               ;
20.0      -90                                               ;
  16        5       6.35                                   ;
16.0       11      3.094                                   ;
 6.0      106       0.25          /SP2/                    ;
16.0       11     3.1523                                   ;
 6.0        1       6.35                                   ;
 6.0      101       6.35                                   ;
 6.0        3       6.35                                   ;
 6.0      103       6.35                                   ;
 2.0   1.1978                     /INP2/                   ;
 4.0  0.91442          0       0  /DP2/                    ;
 2.0   1.1978                     /OUP1/                   ;
20.0       90                                               ;
 3.0    0.203                                               ;
 3.0   0.0254     /W608/                                   ;
 3.0    0.406                                               ;
16.0       11      3.094                                   ;
 6.0      106       0.25       /SP20a/                     ;
16.0       11     3.1523                                   ;
 5.0     0.33     1.7880    4.76 /Q20a/                    ;
16.0       11      3.094                                   ;
 6.0      106       0.25        /SP20/                     ;
16.0       11     3.1523                                   ;
51.0        1        -10      10     0.2  /X20/             ;
52.0        2        -10      10     0.5 /PI20/             ;
51.0        3        -10      10     0.2  /Y20/             ;
52.0        4        -10      10     0.5 /YP20/             ;
50.0       11       3.05     3.2    0.01  /P20/             ;
51.0      101        -10      10     0.2  /X20/             ;
52.0      102        -10      10     0.5 /MU20/             ;
51.0      103        -10      10     0.2  /Y20/             ;
52.0      104        -10      10     0.5 /YP20/             ;
50.0      111       3.05     3.2   0.005  /P20/             ;
50.0      115         -1    -0.8   0.001 /PL20/             ;
 -16      180         51                                   ;
 6.0      106          0          /GMU/                    ;
 6.0        6          0          /GPI/                    ;
 5.0     0.33     1.7880    4.76 /Q20b/                    ;
SENTINEL
SENTINEL
```

Figure 18: Forward 1X



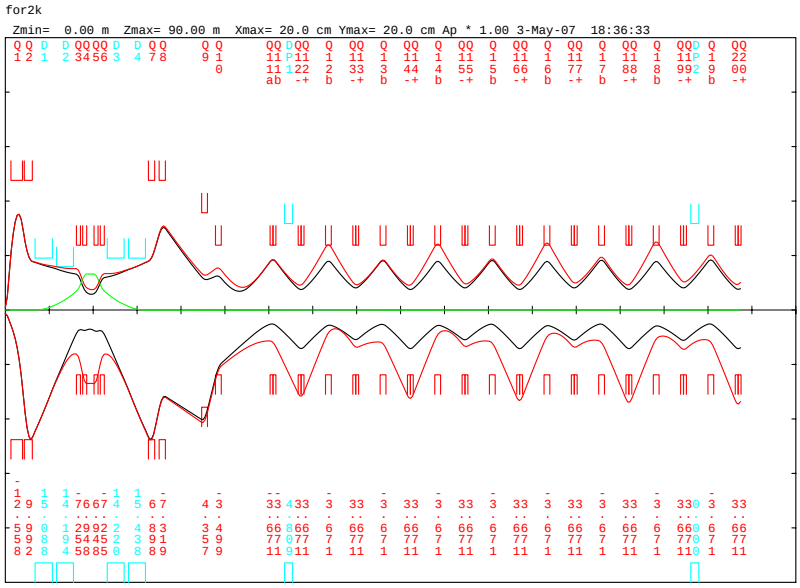
### 7.3.2 Forward 2X

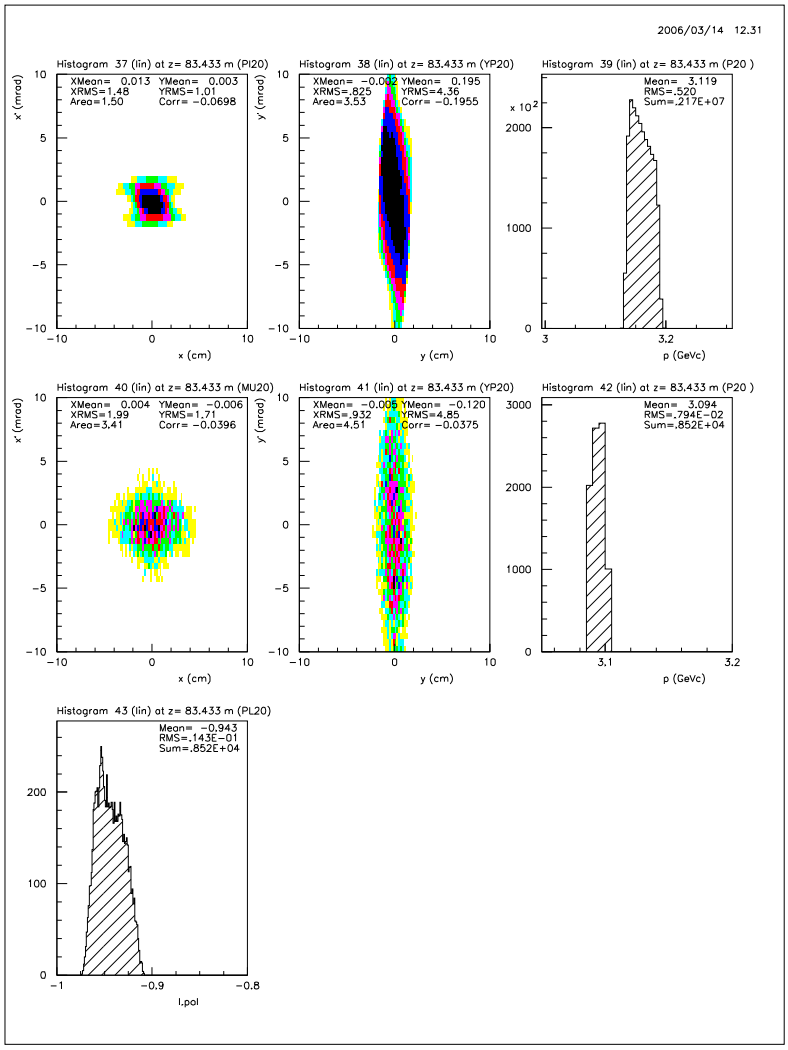
```
/*PLOT*     for2k/
         50000000
( injection scale        1.000        )
(fodo scale              1.000        )
      (                    pi       mu  actual                )
      (         nh          0      100     100                )
      (      p min        5.3     3.05    3.05                )
      (      p max        5.4      3.2     3.2                )
      (     t pion  260000.033   26.033  26.033               )
      ( pitch on/off                        1                 )
      (     PL min         -1   PL max    -0.8                )
      (   Aperture                          1                 )
      (    dP muon                       0.25                 )
      (  transport          )
 (magic      3.094     3.0863    3.0940                       )
     ()
     ()
     ()
     ()
    13.0        10                                                  ;
    13.0       110                                                  ;
    16.0       200    /pion/                                        ;
    16.0       201    /muon/                                        ;
    16.0        20         0                                        ;
     1.0    0.2616        50     0.188     100     8.776     3  3.1523 /pion/;
    60.0    0.1396    0.1057         0  26.033                      ;
   -17.0    /ZWEI/                                                  ;
    16.0       190         0        50                              ;
    51.1         1        -1         1     0.1      /3X/            ;
    52.1         2       -25        25       2    /TARG/            ;
    51.1         3        -1         1     0.1      /Y1/            ;
    52.1         4       -75        75       5    /5V05/            ;
    50.1        11      3.05       3.2   0.005      /P1/            ;
     3.0     0.635                                                  ;
     16        100      9.52      9.52                              ;
     5.0      1.32  -12.5580      9.52    /Q1/                      ;
     ()
     3.0     0.203                                                  ;
     16        100      9.52      9.52                              ;
     5.0     0.914    9.9920      9.52    /Q2/                      ;
     3.0     0.314                                                  ;
    20.0       180                                                  ;
    16.0         5      3.81                                        ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.81                                        ;
     6.0       103      3.81                                        ;
     2.0     8.066     /IN1/                                        ;
     4.0    1.9686   15.0880         0    /D1/                      ;
     2.0     8.066    /OUT1/                                        ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.81                                        ;
     6.0       103      3.81                                        ;
    20.0       180                                                  ;
     3.0     0.478                                                  ;
    20.0       180                                                  ;
    16.0         5      3.18                                        ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.18                                        ;
     6.0       103      3.18                                        ;
     2.0    7.4305     /IN2/                                        ;
     4.0    1.9269   14.1940         0    /D2/                      ;
     2.0    7.4305    /OUT2/                                        ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.18                                        ;
     6.0       103      3.18                                        ;
    20.0       180                                                  ;
     3.0     0.335                                                  ;
     16        100      4.76      4.76                              ;
     5.0     0.457   -7.2550      4.76    /Q3/                      ;
     3.0     0.254                                                  ;
     16        100      4.76      4.76                              ;
     5.0     0.457    6.9480      4.76    /Q4/                      ;
     3.0      0.11                                                  ;
     7.0         1         0         0       0         0       0    ;
     6.0         1      2.66         ;       (     0.812 2.06248    )
     6.0       101      2.66                                        ;
     6.0         3      5.08                                        ;
     6.0       103      5.08                                        ;
     3.0     0.108                                                  ;
     6.0         1      2.53         ;       (      1.14  2.8956    )
     6.0       101      2.53                                        ;
     6.0         3      5.08                                        ;
     6.0       103      5.08                                        ;
     7.0        -1         0         0       0         0       0    ;
     3.0       0.2    /K1K2/                                        ;
    51.0         1        -5         5     0.2   /X2K12/            ;
    52.0         2       -20        20       2    /K1K2/            ;
    51.0         3        -5         5     0.2   /Y2K12/            ;
    52.0         4       -20        20       2   /YPK12/            ;
    51.0        11      3.05       3.2    0.01    /PK12/            ;
    52.0         1        -5         5     0.1    /XK12/            ;
     3.0     0.001                                                  ;
     3.0       0.2                                                  ;
    -6.0         6       0.5                                        ;
     7.0      0.53         0         0       0         0       0    ;
     6.0         1      2.53                                        ;
     6.0       101      2.53                                        ;
     6.0         3      5.08                                        ;
     6.0       103      5.08                                        ;
     3.0     0.108                                                  ;
     6.0         1      2.66                                        ;
     6.0       101      2.66                                        ;
     6.0         3      5.08                                        ;
     6.0       103      5.08                                        ;
     7.0     -0.53         0         0       0         0       0    ;
     3.0      0.11                                                  ;
     16        100      4.76      4.76                              ;
     5.0     0.457    6.9480      4.76    /Q5/                      ;
     3.0     0.254                                                  ;
     16        100      4.76      4.76                              ;
     5.0     0.457   -7.2550      4.76    /Q6/                      ;
     3.0    0.3368                                                  ;
    16.0         5      3.81                                        ;
    20.0       180                                                  ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.81                                        ;
     6.0       103      3.81                                        ;
     2.0    7.4305                          /IN3/                   ;
     4.0    1.9231   14.2200            0   /D3/                    ;
     2.0    7.4305                         /OUT3/                   ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.81                                        ;
     6.0       103      3.81                                        ;
    20.0       180                                                  ;
     3.0     0.503                                                  ;
    20.0       180                                                  ;
    16.0         5      3.81                                        ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.81                                        ;
     6.0       103      3.81                                        ;
     2.0     8.066                          /IN4/                   ;
     4.0    1.9241   15.4380          0   /D4/                      ;
     2.0     8.066                         /OUT4/                   ;
     6.0         1     10.16                                        ;
     6.0       101     10.16                                        ;
     6.0         3      3.81                                        ;
     6.0       103      3.81                                        ;
    20.0       180                                                  ;
     3.0     0.337                                                  ;
     16        100      9.52      9.52                              ;
     5.0     0.711    6.8980      9.52   /Q7/                       ;

     3.0     0.508                                                  ;
     5.0     0.711   -7.3194      9.52   /Q8/                       ;
     3.0      4.13                                                  ;
     16        100      7.14      7.14                              ;
     5.0      0.66    4.3575      7.14   /Q9/                       ;
     16        100      4.76      4.76                              ;
     3.0     0.914                                                  ;
     5.0      0.66   -3.4990      4.76  /Q10/                       ;
     3.0     0.432                                                  ;
     3.0    0.0254                         /W409/                   ;
     3.0     2.318                         /MTCH/                   ;
     3.0     0.001                                                  ;
     3.0    2.7814                                                  ;
    53.1                                                            ;
    16.0        11     3.094                                        ;
     6.0       106      0.25             /SQ11a/                    ;
    16.0        11    3.1523                                        ;
     5.0      0.33   -3.6710      4.76 /Q11a/                       ;
    16.0        11     3.094                                        ;
     6.0       106      0.25              /SQ11/                    ;
    16.0        11    3.1523                                        ;
    51.0       101        -5         5     0.2     /X11/            ;
    52.0       102        -5         5     0.5    /XP11/            ;
    51.0       103        -5         5     0.2     /Y11/            ;
    52.0       104        -5         5     0.5    /YP11/            ;
    51.0       111      3.05       3.2    0.01     /P11/            ;
    52.0       101        -5         5     0.1     /X11/            ;
     5.0      0.33   -3.6710      4.76 /Q11b/                       ;
     3.0     0.432                                                  ;
     3.0    0.0254                         /W430/                   ;
     3.0    0.5334                                                  ;
    20.0        90                                                  ;
    16.0         5      6.35                                        ;
    -6.0         1      12.7                                        ;
    -6.0       101      12.7                                        ;
    -6.0         3      12.7                                        ;
    -6.0       103      12.7                                        ;
    16.0        11     3.094                                        ;
     6.0       106      0.25              /SP1/                     ;
    16.0        11    3.1523                                        ;
     6.0         1      6.35                                        ;
     6.0       101      6.35                                        ;
     6.0         3      6.35                                        ;
     6.0       103      6.35                                        ;
     2.0    1.1978                          /INP1/                  ;
     4.0    0.9142    4.8092            0   /DP1/                   ;
     2.0    1.1978                          /OUP1/                  ;
    20.0       -90                                                  ;
     3.0     0.635                                                  ;
    16.0        11     3.094                                        ;
     6.0       106      0.25            /SQ12a/                     ;
    16.0        11    3.1523                                        ;
     5.0      0.33    3.6710      4.76 /Q12-/                       ;
    16.0        11     3.094                                        ;
     6.0       106      0.25             /SQ12/                     ;
    16.0        11    3.1523                                        ;
    51.0       101        -5         5     0.2     /X12/            ;
    52.0       102        -5         5     0.5    /XP12/            ;
    51.0       103        -5         5     0.2     /Y12/            ;
    52.0       104        -5         5     0.5    /YP12/            ;
    50.0       115        -1      -0.8   0.001    /PL12/            ;
     5.0      0.33    3.6710      4.76 /Q12+/                       ;
     3.0      2.45                                                  ;
     5.0      0.66   -3.6710      4.76 /Q12b/                       ;
     3.0      2.45                                                  ;
    16.0        11     3.094                                        ;
     6.0       106      0.25            /SQ13a/                     ;
    16.0        11    3.1523                                        ;
     5.0      0.33    3.6710      4.76 /Q13-/                       ;
    16.0        11     3.094                                        ;
     6.0       106      0.25             /SQ13/                     ;
    16.0        11    3.1523                                        ;
    51.0       101        -5         5     0.2     /X13/            ;
```

```
52.0      102       -5       5    0.5   /XP13/          ;
51.0      103       -5       5    0.2    /Y13/          ;
52.0      104       -5       5    0.5   /YP13/          ;
50.0      115       -1    -0.8   0.001   /PL13/          ;
 5.0     0.33   3.6710    4.76 /Q13+/                   ;
 3.0     2.45                                            ;
 5.0     0.66  -3.6710    4.76 /Q13b/                   ;
 3.0     2.45                                            ;
16.0       11    3.094                                   ;
 6.0      106     0.25                                   ;
16.0       11   3.1523                                   ;
 6.0        1     6.35                                   ;
 6.0      101     6.35                                   ;
 6.0        3     6.35                                   ;
 6.0      103     6.35                                   ;
 ()                                                      ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SQ14a/                   ;
16.0       11   3.1523                                   ;
 5.0     0.33   3.6710    4.76 /Q14-/                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SQ14/                    ;
16.0       11   3.1523                                   ;
51.0      101       -5       5    0.2    /X14/          ;
52.0      102       -5       5    0.5   /XP14/          ;
51.0      103       -5       5    0.2    /Y14/          ;
52.0      104       -5       5    0.5   /YP14/          ;
50.0      115       -1    -0.8   0.001   /PL14/          ;
 5.0     0.33   3.6710    4.76 /Q14+/                   ;
 3.0     2.45                                            ;
 5.0     0.66  -3.6710    4.76 /Q14b/                   ;
 3.0     2.45                                            ;
 ()
 5.0     0.33   3.6710    4.76 /Q15-/                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SQ15/                    ;
16.0       11   3.1523                                   ;
51.0      101       -5       5    0.2    /X15/          ;
52.0      102       -5       5    0.5   /XP15/          ;
51.0      103       -5       5    0.2    /Y15/          ;
52.0      104       -5       5    0.5   /YP15/          ;
50.0      115       -1    -0.8   0.001    /P15/          ;
 5.0     0.33   3.6710    4.76 /Q15+/                   ;
 3.0     2.45                                            ;
 5.0     0.66  -3.6710    4.76 /Q15b/                   ;
 3.0     2.45                                            ;
 ()
 ()
 5.0     0.33   3.6710    4.76 /Q16-/                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SQ16/                    ;
16.0       11   3.1523                                   ;
51.0      101       -5       5    0.2    /X16/          ;
52.0      102       -5       5    0.5   /XP16/          ;
51.0      103       -5       5    0.2    /Y16/          ;
52.0      104       -5       5    0.5   /YP16/          ;
50.0      115       -1    -0.8   0.001    /P16/          ;
 5.0     0.33   3.6710    4.76 /Q16+/                   ;
 3.0     2.45                                            ;
 5.0     0.66  -3.6710    4.76 /Q16b/                   ;
 3.0     2.45                                            ;
 ()
 5.0     0.33   3.6710    4.76 /Q17-/                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SQ17/                    ;
16.0       11   3.1523                                   ;
51.0      101       -5       5    0.2    /X17/          ;
52.0      102       -5       5    0.5   /XP17/          ;
51.0      103       -5       5    0.2    /Y17/          ;
52.0      104       -5       5    0.5   /YP17/          ;
50.0      115       -1    -0.8   0.001    /P17/          ;
 5.0     0.33   3.6710    4.76 /Q17+/                   ;
 3.0     2.45                                            ;
 5.0     0.66  -3.6710    4.76 /Q17b/                   ;
 3.0     2.45                                            ;
 ()
 5.0     0.33   3.6710    4.76 /Q18-/                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SQ18/                    ;
16.0       11   3.1523                                   ;
51.0      101       -5       5    0.2    /X18/          ;
52.0      102       -5       5    0.5   /XP18/          ;
51.0      103       -5       5    0.2    /Y18/          ;
52.0      104       -5       5    0.5   /YP18/          ;
50.0      115       -1    -0.8   0.001    /P18/          ;
 5.0     0.33   3.6710    4.76 /Q18+/                   ;
 3.0     2.45                                            ;
 5.0     0.66  -3.6710    4.76 /Q18b/                   ;
 3.0     2.45                                            ;
 ()
 5.0     0.33   3.6710    4.76 /Q19-/                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SQ19/                    ;
16.0       11   3.1523                                   ;
51.0        1       -5       5    0.2    /X19/          ;
52.0        2       -5       5    0.5   /PI19/          ;
51.0        3       -5       5    0.2    /Y19/          ;
52.0        4       -5       5    0.5   /YP19/          ;
50.0       11  2.994685 3.309915   0.005    /P19/          ;
51.0      101       -5       5    0.2    /X19/          ;
52.0      102       -5       5    0.5   /MU19/          ;
51.0      103       -5       5    0.2    /Y19/          ;
52.0      104       -5       5    0.5   /YP19/          ;
50.0      115       -1    -0.8   0.001   /PL19/          ;
 5.0     0.33   3.6710    4.76 /Q19+/                   ;
 3.0      0.5                                            ;
20.0      -90                                            ;
  16        5     6.35                                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SP2/                     ;
16.0       11   3.1523                                   ;
 6.0        1     6.35                                   ;
 6.0      101     6.35                                   ;
 6.0        3     6.35                                   ;
 6.0      103     6.35                                   ;
 2.0   1.1978                    /INP2/                  ;
 4.0  0.91442        0        0  /DP2/                   ;
 2.0   1.1978                    /OUP1/                  ;
20.0       90                                            ;
 3.0  1.03558                                            ;
 5.0     0.66  -3.6710    4.76 /Q19b/                   ;
 3.0     2.45                                            ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SP20a/                   ;
16.0       11   3.1523                                   ;
 5.0     0.33   3.6710    4.76 /Q20-/                   ;
16.0       11    3.094                                   ;
 6.0      106     0.25        /SP20/                    ;
16.0       11   3.1523                                   ;
51.0        1      -10      10    0.2    /X20/          ;
52.0        2      -10      10    0.5   /PI20/          ;
51.0        3      -10      10    0.2    /Y20/          ;
52.0        4      -10      10    0.5   /YP20/          ;
50.0       11  2.994685 3.309915   0.005    /P20/          ;
51.0      101      -10      10    0.2    /X20/          ;
52.0      102      -10      10    0.5   /MU20/          ;
51.0      103      -10      10    0.2    /Y20/          ;
52.0      104      -10      10    0.5   /YP20/          ;
50.0      111     3.05     3.2   0.005    /P20/          ;
50.0      115       -1    -0.8   0.001   /PL20/          ;
 -16      180       51                                   ;
 6.0      106        0            /GMU/                  ;
 6.0        6        0            /GPI/                  ;
 5.0     0.33   3.6710    4.76 /Q20+/                   ;
SENTINEL
SENTINEL
```

Figure 19: Forward 2X



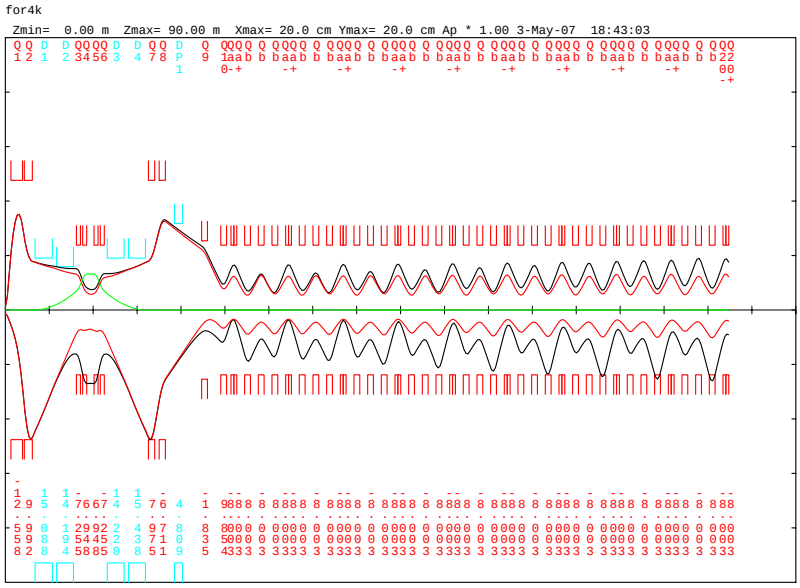



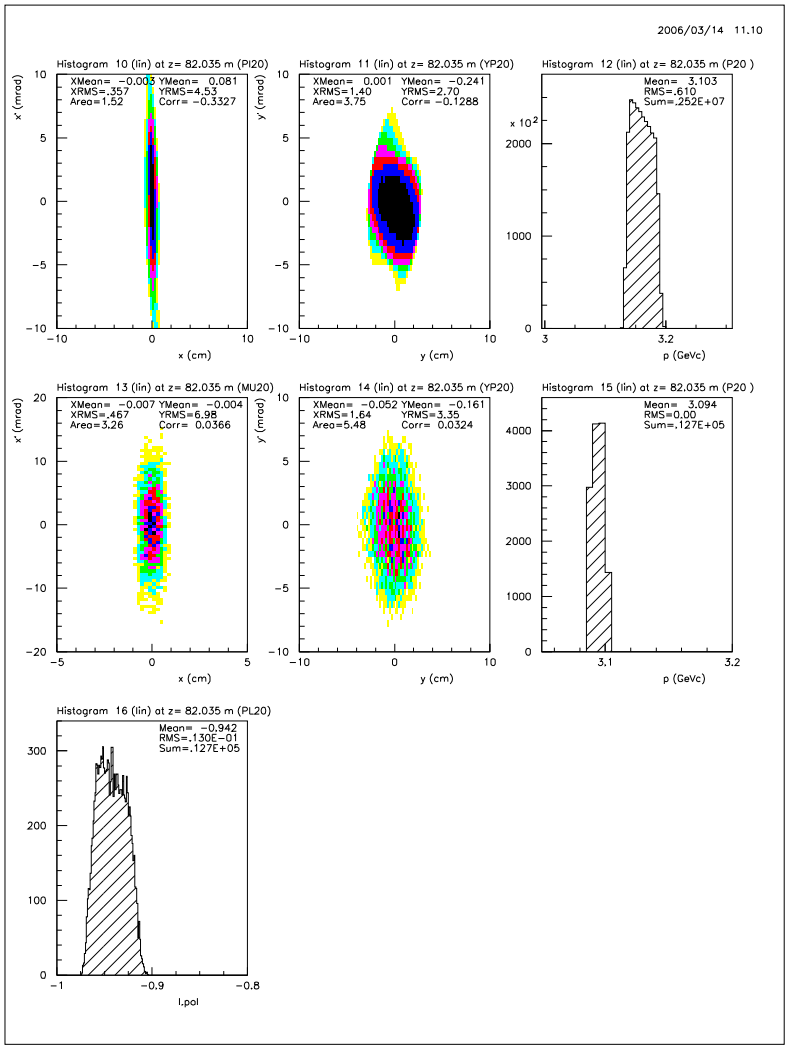
### 7.3.3 Forward 4X

```
      /*PLOT*     for4k/
               50000000
 ( injection scale           1.000      )
  (fodo scale                1.000      )
           (                  pi      mu  actual                  )
           (         nh        0     100     100                  )
           (      p min      5.3    3.05    3.05                  )
           (      p max      5.4     3.2     3.2                  )
           (     t pion  260000.033  26.033  26.033               )
           ( pitch on/off                      1                  )
           (     PL min       -1 PL max     -0.8                  )
           (   Aperture                        1                  )
           (    dP muon                     0.25                  )
           (  transport          )
       (magic       3.094     3.0863  3.0940                      )
           ()
           ()
           ()
           ()
          13.0        10                                                  ;
          13.0       110                                                  ;
          16.0       200   /pion/                                         ;
          16.0       201   /muon/                                         ;
          16.0        20        0                                         ;
           1.0    0.2616       50   0.188     100      8.776       3  3.1523 /pion/;
          60.0    0.1396   0.1057       0  26.033                          ;
         -17.0    /ZWEI/                                                  ;
          16.0       190        0      50                                 ;
          51.1         1       -1       1     0.1      /3X/               ;
          52.1         2      -25      25       2    /TARG/               ;
          51.1         3       -1       1     0.1      /Y1/               ;
          52.1         4      -75      75       5    /5V05/               ;
          50.1        11     3.05     3.2   0.005      /P1/               ;
           3.0     0.635                                                  ;
            16       100     9.52    9.52                                 ;
           5.0      1.32  -12.558    9.52   /Q1/                          ;
           ()
           3.0     0.203                                                  ;
            16       100     9.52    9.52                                 ;
           5.0     0.914    9.992    9.52   /Q2/                          ;
           3.0     0.314                                                  ;
          20.0       180                                                  ;
          16.0         5     3.81                                         ;
           6.0         1    10.16                                         ;
           6.0       101    10.16                                         ;
           6.0         3     3.81                                         ;
           6.0       103     3.81                                         ;
           2.0     8.066   /IN1/                                          ;
           4.0    1.9686   15.088       0   /D1/                          ;
           2.0     8.066   /OUT1/                                         ;
           6.0         1    10.16                                         ;
           6.0       101    10.16                                         ;
           6.0         3     3.81                                         ;
           6.0       103     3.81                                         ;
          20.0       180                                                  ;
           3.0     0.478                                                  ;
          20.0       180                                                  ;
          16.0         5     3.18                                         ;
           6.0         1    10.16                                         ;
           6.0       101    10.16                                         ;
           6.0         3     3.18                                         ;
           6.0       103     3.18                                         ;
           2.0    7.4305   /IN2/                                          ;
           4.0    1.9269   14.194       0   /D2/                          ;
           2.0    7.4305   /OUT2/                                         ;
           6.0         1    10.16                                         ;
           6.0       101    10.16                                         ;
           6.0         3     3.18                                         ;
           6.0       103     3.18                                         ;
          20.0       180                                                  ;
           3.0     0.335                                                  ;
            16       100     4.76    4.76                                 ;
           5.0     0.457   -7.255    4.76   /Q3/                          ;
           3.0     0.254                                                  ;
            16       100     4.76    4.76                                 ;
           5.0     0.457    6.948    4.76   /Q4/                          ;
           3.0      0.11                                                  ;
           7.0         1        0       0       0          0       0      ;
           6.0         1     2.66       ;       (        0.812 2.06248    )
           6.0       101     2.66                                         ;
           6.0         3     5.08                                         ;
           6.0       103     5.08                                         ;
           3.0     0.108                                                  ;
           6.0         1     2.53       ;       (         1.14  2.8956    )
           6.0       101     2.53                                         ;
           6.0         3     5.08                                         ;
           6.0       103     5.08                                         ;
           7.0        -1        0       0       0          0       0      ;
           3.0       0.2   /K1K2/                                         ;
          51.0         1       -5       5     0.2   /X2K12/               ;
          52.0         2      -20      20       2    /K1K2/               ;
          51.0         3       -5       5     0.2   /Y2K12/               ;
          52.0         4      -20      20       2   /YPK12/               ;
          51.0        11     3.05     3.2    0.01    /PK12/               ;
          52.0         1       -5       5     0.1    /XK12/               ;
           3.0     0.001                                                  ;
           3.0       0.2                                                  ;
          -6.0         6      0.5                                         ;
           7.0      0.53        0       0       0          0       0      ;
           6.0         1     2.53                                         ;
           6.0       101     2.53                                         ;
           6.0         3     5.08                                         ;
           6.0       103     5.08                                         ;
           3.0     0.108                                                  ;
           6.0         1     2.66                                         ;
           6.0       101     2.66                                         ;
           6.0         3     5.08                                         ;
           6.0       103     5.08                                         ;
           7.0     -0.53        0       0       0          0       0      ;
           3.0      0.11                                                  ;
            16       100     4.76    4.76                                           ;
           5.0     0.457    6.948    4.76   /Q5/                                    ;
           3.0     0.254                                                            ;
            16       100     4.76    4.76                                           ;
           5.0     0.457   -7.255    4.76   /Q6/                                    ;
           3.0    0.3368                                                            ;
          16.0         5     3.81                                                   ;
          20.0       180                                                            ;
           6.0         1    10.16                                                   ;
           6.0       101    10.16                                                   ;
           6.0         3     3.81                                                   ;
           6.0       103     3.81                                                   ;
           2.0    7.4305                   /IN3/                                    ;
           4.0    1.9231   14.220       0   /D3/                                    ;
           2.0    7.4305                  /OUT3/                                    ;
           6.0         1    10.16                                                   ;
           6.0       101    10.16                                                   ;
           6.0         3     3.81                                                   ;
           6.0       103     3.81                                                   ;
          20.0       180                                                            ;
           3.0     0.503                                                            ;
          20.0       180                                                            ;
          16.0         5     3.81                                                   ;
           6.0         1    10.16                                                   ;
           6.0       101    10.16                                                   ;
           6.0         3     3.81                                                   ;
           6.0       103     3.81                                                   ;
           2.0     8.066                   /IN4/                                    ;
           4.0    1.9241   15.438       0   /D4/                                    ;
           2.0     8.066                  /OUT4/                                    ;
           6.0         1    10.16                                                   ;
           6.0       101    10.16                                                   ;
           6.0         3     3.81                                                   ;
           6.0       103     3.81                                                   ;
          20.0       180                                                            ;
           3.0     0.337                                                            ;
            16       100     9.52    9.52                                           ;
           5.0     0.711    7.975    9.52   /Q7/                                    ;

           3.0     0.508                                                            ;
           5.0     0.711   -6.711    9.52   /Q8/                                    ;
           3.0    1.0308                                                            ;
             6         1     6.35                                                   ;
             6       101     6.35                                                   ;
             6         3     6.35                                                   ;
             6       103     6.35                                                   ;
            20        90                                                            ;
            16         5     6.35                                                   ;
             2    1.1978                  /INP1/                                    ;
             4    0.9142    4.809       0  /DP1/                                    ;
             2    1.1978                  /OUP1/                                    ;
            20       -90                                                            ;
           3.0     2.185                                                        0   ;
            16       100     4.76    4.76                                           ;
           5.0      0.66   -1.835    5.08   /Q9/                                    ;
           3.0    1.5169                                                            ;
           5.0      0.66    9.854    4.76  /Q10/                                    ;
           3.0    0.4754                                                            ;
           3.0    0.0254                                                            ;
           ()
           ()
           ()
          53.1                                                                      ;
          16.0        11    3.094                                                   ;
           6.0       106     0.25         /SQ11a/                                   ;
          16.0        11    3.152                                                   ;
          51.0       101       -5       5     0.2     /X11/                         ;
          52.0       102       -5       5     0.5    /XP11/                         ;
          51.0       103       -5       5     0.2     /Y11/                         ;
          52.0       104       -5       5     0.5    /YP11/                         ;
          50.0       115       -1    -0.8   0.001    /PL11/                         ;
             9         9                                                            ;
           5.0      0.33   -8.003    4.76  /Qa-/                                    ;
          16.0        11    3.094                                                   ;
           6.0       106     0.25            /SQa/                                  ;
          16.0        11    3.152                                                   ;
           5.0      0.33   -8.003    4.76  /Qa+/                                    ;
           3.0     0.896                                                            ;
           5.0      0.66    8.003    4.76   /Qb/                                    ;
           3.0     0.896                                                            ;
           5.0      0.66   -8.003    4.76   /Qb/                                    ;
           3.0     0.896                                                            ;
           5.0      0.66    8.003    4.76   /Qb/                                    ;
           3.0     0.896                                                            ;
           9.0                                                                      ;
          16.0        11    3.094                                                   ;
           6.0       106     0.25          /SP20a/                                  ;
          16.0        11   3.1523                                                   ;
           5.0      0.33   -8.003    4.76 /Q20-/                                    ;
          16.0        11    3.094                                                   ;
           6.0       106     0.25           /SP20/                                  ;
          16.0        11   3.1523                                                   ;
          51.0         1      -10      10     0.2     /X20/                         ;
          52.0         2      -10      10     0.5    /PI20/                         ;
          51.0         3      -10      10     0.2     /Y20/                         ;
          52.0         4      -10      10     0.5    /YP20/                         ;
          50.0        11    2.995   3.310   0.005     /P20/                         ;
          51.0       101       -5       5     0.2     /X20/                         ;
          52.0       102      -20      20     0.5    /MU20/                         ;
          51.0       103      -10      10     0.2     /Y20/                         ;
          52.0       104      -10      10     0.5    /YP20/                         ;
          50.0       111     3.05     3.2   0.005     /P20/                         ;
          50.0       115       -1    -0.8   0.001    /PL20/                         ;
           -16       180       51                                                   ;
           6.0       106        0          /GMU/                                    ;
           6.0         6        0          /GPI/                                    ;
           5.0      0.33   -8.003    4.76 /Q20+/                                    ;
      SENTINEL
      SENTINEL
```

Figure 20: Forward 4X

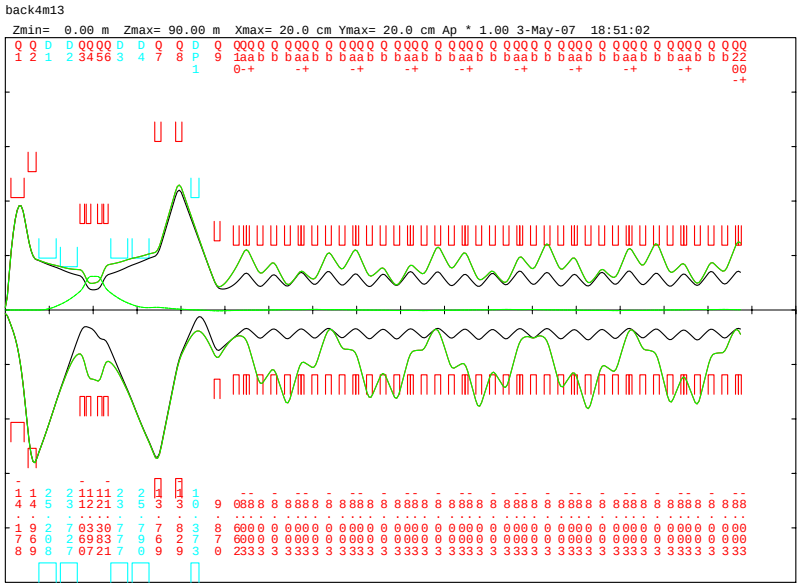


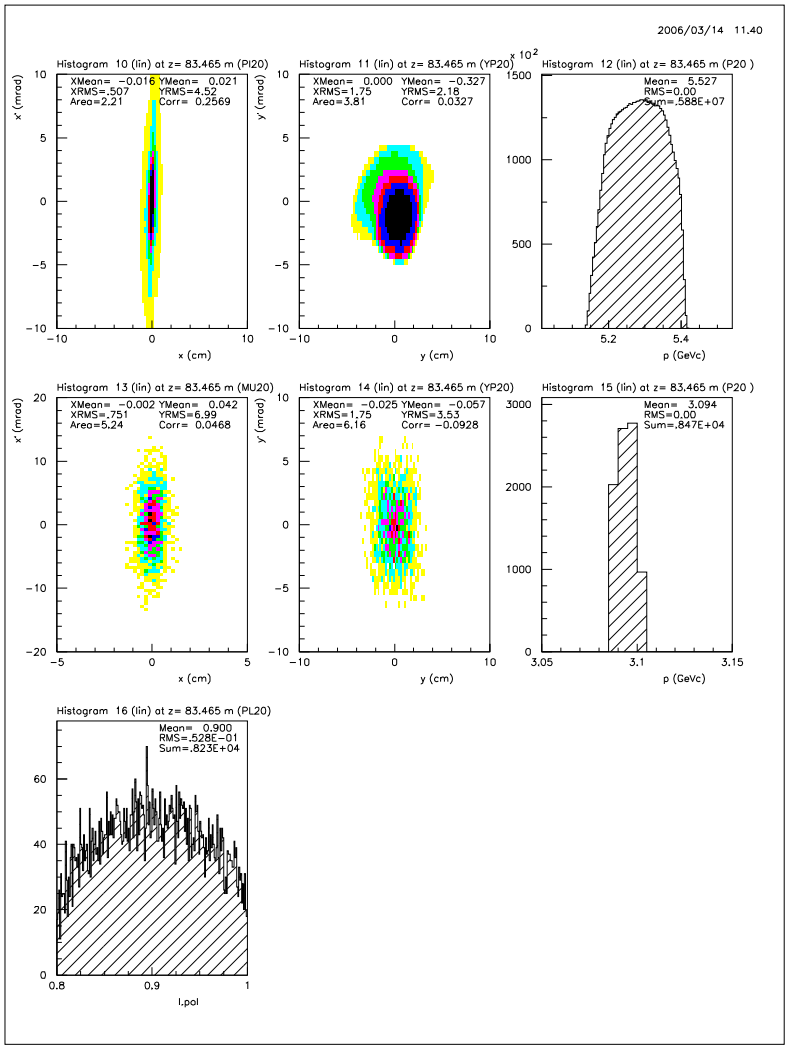



### 7.3.4 Backward 4X

```
     /*PLOT*                    back4m13/
                 50000000
 ( injection scale                 0.987      )
  (fodo scale                      0.987      )
          (                          pi      mu  actual                )
          (             nh            0     100     100                )
          (          p min          5.3    3.05    3.05                )
          (          p max          5.4    3.15    3.15                )
          (         t pion   260000.033  26.033  26.033                )
          (     pitch on/off                          1                )
          (         PL min          0.8 PL max        1                )
          (       Aperture                            1                )
          (        dP muon                         0.25                )
          (      transport             )
      (magic           3.094       3.0863  3.0940                      )
          ()
          ()
          ()
          ()
       13.0             10                                                     ;
       13.0            110                                                     ;
       16.0            200      /pion/                                         ;
       16.0            201      /muon/                                         ;
       16.0             20            0                                        ;
        1.0         0.2616           50   0.188     100     8.776       3   5.280 /pion/;
       60.0         0.1396       0.1057       0  26.033                        ;
      -17.0         /ZWEI/                                                     ;
       16.0            190            0      50                                ;
       51.1              1           -1       1     0.1      /3X/              ;
       52.1              2          -25      25       2    /TARG/              ;
       51.1              3           -1       1     0.1      /Y1/              ;
       52.1              4          -75      75       5    /5V05/              ;
       50.1             11          5.3     5.4   0.005      /P1/              ;
        3.0          0.635                                                     ;
         16            100         7.94    7.94                                ;
        5.0          1.505      -14.178   8.255    /Q1/                        ;
        3.0           0.25                                                     ;
        3.0          0.203                                                     ;
         16            100         9.84    9.84                                ;
        5.0          0.914       14.969   10.16    /Q2/                        ;
        3.0          0.314                                                     ;
       20.0            180                                                     ;
       16.0              5         3.81                                        ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.81                                        ;
        6.0            103         3.81                                        ;
        2.0          8.066      /IN1/                                          ;
        4.0         1.9686       25.208       0    /D1/                        ;
        2.0          8.066      /OUT1/                                         ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.81                                        ;
        6.0            103         3.81                                        ;
       20.0            180                                                     ;
        3.0          0.478                                                     ;
       20.0            180                                                     ;
       16.0              5         3.18                                        ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.18                                        ;
        6.0            103         3.18                                        ;
        2.0         7.4305      /IN2/                                          ;
        4.0         1.9269       23.727       0    /D2/                        ;
        2.0         7.4305      /OUT2/                                         ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.18                                        ;
        6.0            103         3.18                                        ;
       20.0            180                                                     ;
        3.0         0.3032                                                     ;
         16            100         6.03    6.03                                ;
        5.0         0.5208      -11.060    6.35    /Q3/                        ;
        3.0         0.1903                                                     ;
         16            100         6.03    6.03                                ;
        5.0         0.5208       12.397    6.35    /Q4/                        ;
        3.0           0.13                                                     ;
       -7.0          0.246            0       0       0         0       0      ;
       -6.0              1        1.812                                        ;
       -6.0            101        1.812                                        ;
       -6.0              3         5.08                                        ;
       -6.0            103         5.08                                        ;
        3.0         0.0508                                                     ;
       -6.0              1        1.764                                        ;
       -6.0            101        1.764                                        ;
       -6.0              3         5.08                                        ;
       -6.0            103         5.08                                        ;
        3.0          0.202      /K1K2/                                         ;
       51.0              1           -5       5     0.2   /X2K12/              ;
       52.0              2          -20      20       2    /K1K2/              ;
       51.0              3           -5       5     0.2   /Y2K12/              ;
       52.0              4          -20      20       2   /YPK12/              ;
       51.0             11          5.3     5.4    0.01    /PK12/              ;
       52.0              1           -5       5     0.1    /XK12/              ;
        3.0          0.001                                                     ;
       -7.0         -0.246            0       0       0         0       0      ;
        3.0          0.001                                                     ;
        3.0          0.202                                                     ;
       -7.0         -0.246            0       0       0         0       0      ;
       -6.0              1        1.764                                        ;
       -6.0            101        1.764                                        ;
       -6.0              3         5.08                                        ;
       -6.0            103         5.08                                        ;
        3.0         0.0508                                                     ;
       -6.0              1        1.812                                        ;
       -6.0            101        1.812                                        ;
       -6.0              3         5.08                                        ;
       -6.0            103         5.08                                        ;
       -7.0          0.246            0       0       0         0       0      ;
        3.0           0.13                                                     ;
         16            100         6.03    6.03                                ;
        5.0          0.508       12.382    6.35    /Q5/                        ;
        3.0         0.1905                                                     ;
         16            100         6.03    6.03                                ;
        5.0          0.508      -11.031    6.35    /Q6/                        ;
        3.0        0.30495                                                     ;
       16.0              5         3.81                                        ;
       20.0            180                                                     ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.81                                        ;
        6.0            103         3.81                                        ;
        2.0         7.4305                        /IN3/                        ;
        4.0         1.9231       23.777       0    /D3/                        ;
        2.0         7.4305                        /OUT3/                       ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.81                                        ;
        6.0            103         3.81                                        ;
       20.0            180                                                     ;
        3.0          0.503                                                     ;
       20.0            180                                                     ;
       16.0              5         3.81                                        ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.81                                        ;
        6.0            103         3.81                                        ;
        2.0          8.066                        /IN4/                        ;
        4.0         1.9241       25.790       0    /D4/                        ;
        2.0          8.066                        /OUT4/                       ;
        6.0              1        10.16                                        ;
        6.0            101        10.16                                        ;
        6.0              3         3.81                                        ;
        6.0            103         3.81                                        ;
       20.0            180                                                     ;
        3.0          0.652                           ;          (       1.3      )
         16            100       12.792  12.792                                ;
        5.0          0.711       13.769  12.376    /Q7/                        ;
        ()
        3.0          1.669                                                     ;
        5.0          0.711      -13.829  12.376    /Q8/                        ;
        3.0         1.0308                                                     ;
          6              1        8.255                                        ;
          6            101        8.255                                        ;
          6              3        8.255                                        ;
          6            103        8.255                                        ;
         20             90                                                     ;
         16              5        8.255                                        ;
          2         1.1978                        /INP1/                       ;
          4         0.9142       10.373   0.000   /DP1/                        ;
          2         1.1978                        /OUP1/                       ;
         20            -90                                                     ;
        3.0          1.734                                                     ;
         16            100         4.76    4.76                                ;
        5.0           0.66         9.87    5.08    /Q9/                        ;
        3.0         1.5169                                                     ;
        5.0           0.66        0.662    4.76   /Q10/                        ;
        3.0         0.4754                                                     ;
        3.0         0.0254                                                     ;
        ()
        ()
        ()
       53.1                                                                    ;
       16.0             11        3.094                                        ;
        6.0            106         0.25         /SQ11a/                        ;
       16.0             11        5.280                                        ;
       51.0            101           -5       5     0.2     /X11/              ;
       52.0            102           -5       5     0.5    /XP11/              ;
       51.0            103           -5       5     0.2     /Y11/              ;
       52.0            104           -5       5     0.5    /YP11/              ;
       50.0            115          0.8       1   0.001    /PL11/              ;
          9              9                                                     ;
        5.0           0.33       -8.003    4.76   /Qa-/                        ;
       16.0             11        3.094                                        ;
        6.0            106         0.25           /SQa/                        ;
       16.0             11        5.280                                        ;
        5.0           0.33       -8.003    4.76   /Qa+/                        ;
        3.0          0.896                                                     ;
        5.0           0.66        8.003    4.76    /Qb/                        ;
        3.0          0.896                                                     ;
        5.0           0.66       -8.003    4.76    /Qb/                        ;
        3.0          0.896                                                     ;
        5.0           0.66        8.003    4.76    /Qb/                        ;
        3.0          0.896                                                     ;
        9.0                                                                    ;
       16.0             11        3.094                                        ;
        6.0            106         0.25         /SP20a/                        ;
       16.0             11        5.280                                        ;
        5.0           0.33       -8.003    4.76 /Q20-/                         ;
       16.0             11        3.094                                        ;
        6.0            106         0.25          /SP20/                        ;
       16.0             11        5.280                                        ;
       51.0              1          -10      10     0.2     /X20/              ;
       52.0              2          -10      10     0.5    /PI20/              ;
       51.0              3          -10      10     0.2     /Y20/              ;
       52.0              4          -10      10     0.5    /YP20/              ;
       50.0             11        5.016   5.544   0.005     /P20/              ;
       51.0            101           -5       5     0.2     /X20/              ;
       52.0            102          -20      20     0.5    /MU20/              ;
       51.0            103          -10      10     0.2     /Y20/              ;
       52.0            104          -10      10     0.5    /YP20/              ;
       50.0            111         3.05    3.15   0.005     /P20/              ;
       50.0            115          0.8       1   0.001    /PL20/              ;
        -16            180           51                                        ;
        6.0            106            0           /GMU/                        ;
        6.0              6            0           /GPI/                        ;
        5.0           0.33       -8.003    4.76 /Q20+/                         ;
   SENTINEL
   SENTINEL
```

Figure 21: Backward 4X



### 7.3.5 Achromat

```
/g2-achromat3/
50000000
16. 190. 0. 50. ;
13. 10.0 ;
13. 110.0 ;
16. 200. /pion/ ;
16. 201. /muon/ ;
16. 202. /nu/ ;
16. 20. ;
(envelope for beam)
1. /g2/ .2616 50 .188 100 0.0 3 5.236 ;
60. 0.13957018 0.105658357 0.0 26.033 ;
17. ;
(histos 1-6)
50.0 6 -6.06 6.06 .12 ;
50.1 6 -6.06 6.06 .12 ;
50.2 6 -6.06 6.06 .12 ;
50.3 6 -6.06 6.06 .12 ;
50.4 6 -6.06 6.06 .12 ;
50.5 6 -6.06 6.06 .12 ;

3. 0.5027 ;
5. 1.504 -14.4119 8.255 /Qm11/ ;
3. 0.5431 /1/ ;
5. 0.914 14.8485 10.16 /Qm12/ ;
3. 4.132 /2/ ;
5. 0.66 -9.2213 8.0 /Qm13/ ;
3. 1.408 /3/ ;
5. 0.66 12.6525 8.0 /Qm14/ ;
3. 0.7378 /4/ ;
5. 0.33 9.37863 8.0 /Qm15/ ;

(flag pions entering achromat)
53.1 ;

(the achromat)
9. 4.0 ;
5. 0.33 9.37863 8.0 /Q1a/ ;
16. 100. 8. 8. ;
18. 0.2 0.58017 8.0 /S1/ ;
3. 0.0706 ;
4. 1.006 23.4934 0.0 /D1/ ;
3. 1.1162 ;
18. 0.2 -0.99002 8.0 /S2/ ;
5. 0.66 -9.80178 8.0 /Q2/ ;
18. 0.2 -0.99002 8.0 /S3/ ;
3. 1.1162 ;
4. 1.006 23.4934 0.0 /D2/ ;
3. 0.0706 ;
18. 0.2 0.58017 8.0 /S4/ ;
5. 0.33 9.37863 8.0 /Q1b/ ;
9. 0.0 ;

(flag pions exiting achromat)
53.2 ;

(matching into decay lattice)
5. 0.33 9.37863 8.0 /Qm21/ ;
3. 0.3 ;
5. 0.66 -9.26524 8.0 /Qm22/ ;
3. 0.1 ;
5. 0.66 12.8033 8.0 /Qm23/ ;
3. 1.203 ;
16. 100. 4.76 4.76 ;
5. 0.66 -7.6496 4.76 /Qm24/ ;
3. 1.4265 ;
5. 0.66 9.53181 4.76 /Qm25/ ;
3. 0.3 ;
5. .33 -8.0 4.76 /Qm26/ ;

(flag pions which enter the lattice)
53.3 ;
16. 11. 3.094 ;
6. 106. 0.5 ;

(lattice, 80 m long)
9. 18. ;
    5. .33 -8.0 4.76 /LQ1a/ ;
    3. .896 ;
    5. .66 8.0 4.76 /LQ2/ ;
    3. .896 ;
    5. .33 -8.0 4.76 /LQ1b/ ;
    6. 106. 0.5 ;
9. ;

(pions which exit lattice)
53.4 ;

(histo 7)
50.0 106 -.7575 .7575 .015 ;
(histo 8)
50.0 106 -.505 .505 .01 ;
(histo 9)
50.0 111 2.0 4.0 .05 ;
(histo 10)
50.0 111 2.0 6.0 .05 ;
(histo 11)
51.0 101. -5.0 5.0 .1 ;
52.0 103. -5.0 5.0 .1 ;
(histo 12)
50. 8. 0.0 170.0 1.0 ;
(histo 13)
50. 108. 0.0 170.0 1.0 ;
(select momentum for storable magic muons)
6. 106. 0.25 ;

(histo 14)
50.0 106 -.50 .50 .01 ;
(histo 15)
50.0 115 1.0 .50 .01 ;
(histo 16)
51.0 101. -5.00 5.00 .1 ;
52.0 103. -5.00 5.00 .1 ;
(histo 17)
51.0 115 1. .7 .01 ;
52.0 106 -.50 .50 .01 ;
(histo 18)
51.0 106 -.50 .50 .01 ;
52.0 115 1. .7 .01 ;
(histo 19)
51.0 101. -5.0 5.0 .1 ;
52.0 102. -12.0 12.0 .24 ;
(histo 20)
51.0 103. -5.0 5.0 .1 ;
52.0 104. -10.0 10.0 .2 ;
(histo 21-22)
50. 102. -12.0 12.0 .24 ;
50. 104. -10.0 10.0 .2 ;
SENTINEL
SENTINEL
```

### 7.3.6 Muon Accumulator Ring

```
 /*PLOT*        mar/
            50000000
 ( injection scale            0.993       )
 (fodo scale                  0.993       )
      (                        pi       mu   actual          )
      (          nh          0.000  100.000  100.000         )
      (       p min          5.300    3.050    3.050         )
      (       p max          5.400    3.200    3.200         )
      (      t pion      260000.033   26.033   26.033         )
      ( pitch on/off                   0.000                 )
      (      PL min         -1.000   PL max   -0.800         )
      (    Aperture                    1.000                 )
      (     dP muon                    0.250                 )
      (     dP pion                    3.000                 )
      (    transport            )
  (magic        3.094         3.086    3.094                 )
     ()
     ()
     ()
     ()
    13.0          10                                          ;
    13.0         110                                          ;
    16.0         200       /pion/                             ;
    16.0         201       /muon/                             ;
    16.0          20         0.000                            ;
(    1.0      0.2616        15.000    0.188   60.000      0 0.5  3.1306 /pion/;)
     1.0      0.2616        50.000    0.188  100.000      0   3  3.1306 /pion/;
    60.0      0.1396         0.106    0.000   26.033                ;
   -17.0     /ZWEI/                                            ;
    16.0         190         0.000   50.000                   ;
    51.1           1        -1.000    1.000    0.100   /3X/   ;
    52.1           2       -25.000   25.000    2.000 /TARG/   ;
    51.1           3        -1.000    1.000    0.100   /Y1/   ;
    52.1           4       -75.000   75.000    5.000 /5V05/   ;
    50.1          11         3.050    3.200    0.005   /P1/   ;
     3.0       0.635                                           ;
      16         100         9.520    9.520                   ;
     5.0        1.32       -12.472    9.520    /Q1/           ;
     ()
     3.0       0.203                                           ;
      16         100         9.520    9.520                   ;
     5.0       0.914         9.923    9.520    /Q2/           ;
     3.0       0.314                                           ;
    20.0         180                                           ;
    16.0           5         3.810                            ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.810                            ;
     6.0         103         3.810                            ;
     2.0       8.066        /IN1/                             ;
     4.0      1.9686        14.984    0.000    /D1/           ;
     2.0       8.066       /OUT1/                             ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.810                            ;
     6.0         103         3.810                            ;
    20.0         180                                           ;
     3.0       0.478                                           ;
    20.0         180                                           ;
    16.0           5         3.180                            ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.180                            ;
     6.0         103         3.180                            ;
     2.0      7.4305        /IN2/                             ;
     4.0      1.9269        14.096    0.000    /D2/           ;
     2.0      7.4305       /OUT2/                             ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.180                            ;
     6.0         103         3.180                            ;        (
    20.0         180                                           ;        (
     3.0       0.335                                           ;        (
      16         100         4.760    4.760                   ;        (
     5.0       0.457        -7.205    4.760    /Q3/           ;        (
     3.0       0.254                                           ;        (
      16         100         4.760    4.760                   ;
     5.0       0.457         6.900    4.760    /Q4/           ;
     3.0        0.16                                           ;
    -7.0       0.246         0.000    0.000    0.000      0   0  ;
    -6.0           1         1.812                            ;
    -6.0         101         1.812                            ;
    -6.0           3         5.080                            ;
    -6.0         103         5.080                            ;
     3.0      0.0508                                           ;
    -6.0           1         1.764                            ;
    -6.0         101         1.764                            ;
    -6.0           3         5.080                            ;
    -6.0         103         5.080                            ;
     3.0       0.202       /K1K2/                             ;
(   51.0           1        -5.000    5.000    0.200/X2K12/   ;        )
(   52.0           2       -20.000   20.000    2.000 /K1K2/   ;        )
(   51.0           3        -5.000    5.000    0.200/Y2K12/   ;        )
(   52.0           4       -20.000   20.000    2.000/YPK12/   ;        )
(   51.0          11         3.050    3.200    0.010 /PK12/   ;        )
(   52.0           1        -5.000    5.000    0.100 /XK12/   ;        )
     3.0      0.001                                           ;
    -7.0      -0.246         0.000    0.000    0.000      0   0  ;
     3.0       0.203                                           ;
     6.0           6         3.000                            ;
    -7.0      -0.246         0.000    0.000    0.000      0   0  ;
    -6.0           1         1.764                            ;
    -6.0         101         1.764                            ;
    -6.0           3         5.080                            ;
    -6.0         103         5.080                            ;
     3.0      0.0508                                           ;        (
    -6.0           1         1.812                            ;        (
    -6.0         101         1.812                            ;        (
    -6.0           3         5.080                            ;        (
    -6.0         103         5.080                            ;        (
    -7.0       0.246         0.000    0.000    0.000      0   0  ;
     3.0        0.16                                           ;
      16         100         6.030    4.760                   ;
     5.0       0.457         6.900    4.760    /Q5/           ;
     3.0       0.254                                           ;
      16         100         6.030    4.760                   ;
     5.0       0.457        -7.205    4.760    /Q6/           ;
     3.0      0.3368                                           ;
    16.0           5         3.810                            ;
    20.0         180                                           ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.810                            ;
     6.0         103         3.810                            ;
     2.0      7.4305                        /IN3/             ;
     4.0      1.9231        14.122    0.000    /D3/           ;
     2.0      7.4305                       /OUT3/             ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.810                            ;
     6.0         103         3.810                            ;
    20.0         180                                           ;
     3.0       0.503                                           ;
    20.0         180                                           ;
    16.0           5         3.810                            ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.810                            ;
     6.0         103         3.810                            ;
     2.0       8.066                        /IN4/             ;
     4.0      1.9241        15.332    0.000    /D4/           ;
     2.0       8.066                       /OUT4/             ;
     6.0           1        10.160                            ;
     6.0         101        10.160                            ;
     6.0           3         3.810                            ;
     6.0         103         3.810                            ;
    20.0         180                                           ;
     3.0       0.337                                           ;
      16         100         9.520    9.520                   ;
     5.0       0.711         6.851    9.520    /Q7/           ;

     3.0       0.508                                           ;
     5.0       0.711        -7.269    9.520    /Q8/           ;
     3.0        4.13                                           ;
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
     ()
      16         100         7.140    7.140                   ;
     5.0        0.66         4.328    7.140    /Q9/           ;
      16         100         4.760    4.760                   ;
     3.0       0.914                                           ;
     5.0        0.66        -3.475    4.760   /Q10/           ;
     3.0       0.432                                           ;
     3.0      0.0254                       /W409/             ;
     3.0       2.318                       /MTCH/             ;
     3.0       0.001                                           ;
     3.0      2.7814                                           ;
    53.1                                                       ;
    16.0          11         3.094                            ;
     6.0         106         0.250          /SQ11a/           ;
    16.0          11         3.131                            ;
     5.0        0.33        -3.646    4.760  /Q11a/           ;
    16.0          11         3.094                            ;
     6.0         106         0.250           /SQ11/           ;
    16.0          11         3.131                            ;
    51.0         101        -5.000    5.000    0.200  /X11/   ;        )
    52.0         102        -5.000    5.000    0.500 /XP11/   ;        )
    51.0         103        -5.000    5.000    0.200  /Y11/   ;        )
    52.0         104        -5.000    5.000    0.500 /YP11/   ;        )
    51.0         111         3.050    3.200    0.010  /P11/   ;        )
    52.0         101        -5.000    5.000    0.100  /X11/   ;        )
     5.0        0.33        -3.646    4.760  /Q11b/           ;
     3.0       0.432                                           ;
     3.0      0.0254                       /W430/             ;
     3.0      0.5334                                           ;
    20.0          90                                           ;
    16.0           5         6.350                            ;
    -6.0           1        12.700                            ;
    -6.0         101        12.700                            ;
    -6.0           3        12.700                            ;
    -6.0         103        12.700                            ;
    16.0          11         3.094                            ;
     6.0         106         0.250           /SP1/            ;
    16.0          11         3.131                            ;
     6.0           1         6.350                            ;
     6.0         101         6.350                            ;
     6.0           3         6.350                            ;
     6.0         103         6.350                            ;
     2.0      1.1978                       /INP1/             ;
     4.0      0.9142        0.000     0.000   /DP1/           ;
     2.0      1.1978                       /OUP1/             ;
    20.0         -90                                           ;
     3.0       0.635                                           ;
    16.0          11         3.094                            ;
     6.0         106         0.250         /SQ12a/            ;
    16.0          11         3.131                            ;
     5.0        0.33         3.646    4.760  /Q12-/           ;
    16.0          11         3.094                            ;
     6.0         106         0.250          /SQ12/            ;
    16.0          11         3.131                            ;
    51.0         101        -5.000    5.000    0.200  /X12/   ;        )
    52.0         102        -5.000    5.000    0.500 /XP12/   ;        )
    51.0         103        -5.000    5.000    0.200  /Y12/   ;        )
    52.0         104        -5.000    5.000    0.500 /YP12/   ;        )
    50.0         115        -1.000   -0.800    0.001 /PL12/   ;        )
```

```
      5.0       0.33      3.646    4.760  /Q12+/                 ;
      3.0       2.45                                             ;
      5.0       0.66     -3.646    4.760  /Q12b/                 ;
      3.0       2.45                                             ;
     16.0         11      3.094                                  ;
      6.0        106      0.250         /SQ13a/                  ;
     16.0         11      3.131                                  ;
      5.0       0.33      3.646    4.760  /Q13-/                 ;
     16.0         11      3.094                                  ;
      6.0        106      0.250          /SQ13/                  ;
     16.0         11      3.131                                  ;
(    51.0        101     -5.000    5.000    0.200  /X13/         ;
(    52.0        102     -5.000    5.000    0.500 /XP13/         ;
(    51.0        103     -5.000    5.000    0.200  /Y13/         ;
(    52.0        104     -5.000    5.000    0.500 /YP13/         ;
(    50.0        115     -1.000   -0.800    0.001 /PL13/         ;
      5.0       0.33      3.646    4.760  /Q13+/                 ;
      3.0       2.45                                             ;
      5.0       0.66     -3.646    4.760  /Q13b/                 ;
      3.0       2.45                                             ;
     16.0         11      3.094                                  ;
      6.0        106      0.250                                  ;
     16.0         11      3.131                                  ;
      6.0          1      6.350                                  ;
      6.0        101      6.350                                  ;
      6.0          3      6.350                                  ;
      6.0        103      6.350                                  ;
      ()
     16.0         11      3.094                                  ;
      6.0        106      0.250         /SQ14a/                  ;
     16.0         11      3.131                                  ;
      5.0       0.33      3.646    4.760  /Q14-/                 ;
     16.0         11      3.094                                  ;
      6.0        106      0.250          /SQ14/                  ;
     16.0         11      3.131                                  ;
(    51.0        101     -5.000    5.000    0.200  /X14/         ;
(    52.0        102    -20.000   20.000    0.500 /XP14/         ;
(    51.0        103     -5.000    5.000    0.200  /Y14/         ;
(    52.0        104    -20.000   20.000    0.500 /YP14/         ;
(    50.0        115     -1.000   -0.800    0.001 /PL14/         ;
      5.0       0.33      3.646    4.760  /Q14+/                 ;
      3.0       2.45                                             ;
      5.0       0.66     -3.646    4.760  /Q14b/                 ;
      3.0       2.45                                             ;
      ()
      5.0       0.33      3.646    4.760  /Q15-/                 ;
     16.0         11      3.094                                  ;
      6.0        106      0.250          /SQ15/                  ;
     16.0         11      3.131                                  ;
(    51.0        101     -5.000    5.000    0.200  /X15/         ;
(    52.0        102     -5.000    5.000    0.500 /XP15/         ;
(    51.0        103     -5.000    5.000    0.200  /Y15/         ;
(    52.0        104     -5.000    5.000    0.500 /YP15/         ;
(    50.0        115     -1.000   -0.800    0.001  /P15/         ;
      5.0       0.33      3.646    4.760  /Q15+/                 ;
      3.0       2.45                                             ;
      5.0       0.66     -3.646    4.760  /Q15b/                 ;
      3.0       2.45                                             ;
      ()
      ()
      5.0       0.33      3.646    4.760  /Q16-/                 ;
     16.0         11      3.094                                  ;
      6.0        106      0.250          /SQ16/                  ;
     16.0         11      3.131                                  ;
(    51.0        101     -5.000    5.000    0.200  /X16/         ;
(    52.0        102     -5.000    5.000    0.500 /XP16/         ;
(    51.0        103     -5.000    5.000    0.200  /Y16/         ;
(    52.0        104     -5.000    5.000    0.500 /YP16/         ;
(    50.0        115     -1.000   -0.800    0.001  /P16/         ;
      5.0       0.33      3.646    4.760  /Q16+/                 ;
      3.0       2.45                                             ;
      5.0       0.66     -3.646    4.760  /Q16b/                 ;
      3.0       2.45                                             ;
      ()
      5.0       0.33      3.646    4.760  /Q17-/                 ;
     16.0         11      3.094                                  ;
      6.0        106      0.250          /SQ17/                  ;
     16.0         11      3.131                                  ;
(    51.0        101     -5.000    5.000    0.200  /X17/         ;
(    52.0        102     -5.000    5.000    0.500 /XP17/         ;
(    51.0        103     -5.000    5.000    0.200  /Y17/         ;
(    52.0        104     -5.000    5.000    0.500 /YP17/         ;
(    50.0        115     -1.000   -0.800    0.001  /P17/         ;
      5.0       0.33      3.646    4.760  /Q17+/                 ;
      3.0       2.45                                             ;
      5.0       0.66     -3.646    4.760  /Q17b/                 ;
      3.0       2.45                                             ;
      ()
      5.0       0.33      3.646    4.760  /Q18-/                 ;
     16.0         11      3.094                                  ;
      6.0        106      0.250          /SQ18/                  ;
     16.0         11      3.131                                  ;
(    51.0        101     -5.000    5.000    0.200  /X18/         ;
(    52.0        102     -5.000    5.000    0.500 /XP18/         ;
(    51.0        103     -5.000    5.000    0.200  /Y18/         ;
(    52.0        104     -5.000    5.000    0.500 /YP18/         ;
(    50.0        115     -1.000   -0.800    0.001  /P18/         ;
      5.0       0.33      3.646    4.760  /Q18+/                 ;
      3.0       2.45                                             ;
      5.0       0.66     -3.646    4.760  /Q18b/                 ;
      3.0       2.45                                             ;
  (matching)
      3.0     0.1256                                             ;
      5.0    0.20434      9.895    8.000              /Q0/       ;
     16.0         11      3.094                                  ;
      6.0        106      0.250                     /SQ18/       ;
     16.0         11      3.131                                  ;
     51.0          1     -5.000    5.000    0.200  /Xin/         ;
     52.0          2    -20.000   20.000    1.000 /PIin/         ;
     51.0          3     -5.000    5.000    0.200  /Yin/         ;
     52.0          4    -20.000   20.000    1.000 /YPin/         ;
     50.0         11      2.974    3.287    0.005  /Pin/         ;
```

```
     51.0        101     -5.000    5.000    1.000  /Xin/        ;
     52.0        102    -20.000   20.000    0.500 /MUin/        ;
     51.0        103     -5.000    5.000    1.000  /Yin/        ;
     52.0        104    -20.000   20.000    0.500 /YPin/        ;
     50.0        115     -1.000   -0.800    0.001 /PLin/        ;
(    50.0          5    -50.000   50.000    0.500  /Lin/        ;      )
        3          0                              /TU1/         ;
  (begin   racetrack)
      9.0          2                                            ;
     16.0         11      3.094                                 ;
      6.0        106      0.250                   /SQin/        ;
)    16.0         11      3.131                                 ;
)          (beginning      180.000degree    double ac       )
)       3          0                                            ;
)       9          8                                            ;
)      16        100      8.000    8.000                        ;
        5       0.33      9.895    8.000          /Q1a/         ;
        3       0.03                                            ;
       16        100      8.000    8.000                        ;
       18        0.2      0.844    8.000          /S1a/         ;
        3       0.03                                            ;
        2    10.3055       /1/                                  ;
        4     0.8728     23.492    0.000          /D1a/         ;
        2   -4.97939       /2/                                  ;
        3       0.03                                            ;
       18        0.2     -1.369    8.000          /S2a/         ;
        3       0.03                                            ;
        5       0.66     -9.895    8.000          /Q2a/         ;
        3       0.03                                            ;
       18        0.2     -1.369    8.000          /S2b/         ;
        3       0.03                                            ;
        2   -4.97939       /3/                                  ;
        4     0.8728     23.492    0.000          /D1b/         ;
        2    10.3055       /4/                                  ;
        3       0.03                                            ;
)      18        0.2      0.844    8.000          /S1b/         ;
)       3       0.03                                            ;
)       5       0.33      9.846    8.000          /Q1b/         ;
)                  9                                            ;
)       3          0                                            ;
      5.0    0.20434      9.895    8.000           /Q0/         ;
      3.0     0.1256                                            ;
      3.0       2.45                                            ;
     16.0        100      4.760    4.760                        ;
      5.0       0.33     -3.646    4.760          /F1a/         ;
  (lattice)
        9      5.000                                            ;
        5      0.330     -3.646    4.760          /F1b/         ;
        3      2.450                                            ;
)       5      0.660      3.646    4.760           /F2/         ;
)       3      2.450                                            ;
)       5      0.330     -3.646    4.760          /F3b/         ;
)      16     11.000      3.094                                 ;
)       6    106.000      0.250                  /SQ18/         ;
       16     11.000      3.131                                 ;
        9                                                       ;
      5.0       0.33     -3.646    4.760          /F1a/         ;
      3.0       2.45                                            ;
      3.0     0.1256                                            ;
     16.0        100      8.000    8.000                        ;
      5.0    0.20434      9.895    8.000                        ;
     16.0         11      3.094                                 ;
      6.0        106      0.250                   /SQn/         ;
     16.0         11      3.131                                 ;
)     9.0                                                       ;
)    51.0          1     -5.000    5.000   0.200   /X1/         ;
)    52.0          2    -20.000   20.000   1.000   /P1/         ;
)    51.0          3     -5.000    5.000   0.200   /Y1/         ;
)    52.0          4    -20.000   20.000   1.000  /YP1/         ;
     50.0         11      2.974    3.287   0.005   /P1/         ;
     51.0        101     -5.000    5.000   0.200   /X1/         ;
     52.0        102    -20.000   20.000   1.000  /MU1/         ;
     51.0        103     -5.000    5.000   0.200   /Y1/         ;
     52.0        104    -20.000   20.000   1.000  /YP1/         ;
     50.0        115     -1.000   -0.800   0.001  /PL1/         ;
(    50.0          5    -50.000   50.000   0.500 /Lout/         ;      )
    (end        turn          ,   repeat       n  times)
      (            2          )
)       3          0                              /TU2/         ;
)  (begin   racetrack)
)     9.0          2                                            ;
)    16.0         11      3.094                                 ;
)     6.0        106      0.250                   /SQin/        ;
     16.0         11      3.131                                 ;
           (beginning      180.000degree    double ac       )
        3          0                                            ;
        9          8                                            ;
       16        100      8.000    8.000                        ;
        5       0.33      9.895    8.000          /Q1a/         ;
        3       0.03                                            ;
       16        100      8.000    8.000                        ;
       18        0.2      0.844    8.000          /S1a/         ;
)       3       0.03                                            ;
)       2    10.3055       /1/                                  ;
)       4     0.8728     23.492    0.000          /D1a/         ;
)       2   -4.97939       /2/                                  ;
)       3       0.03                                            ;
       18        0.2     -1.369    8.000          /S2a/         ;
        3       0.03                                            ;
        5       0.66     -9.895    8.000          /Q2a/         ;
        3       0.03                                            ;
       18        0.2     -1.369    8.000          /S2b/         ;
        3       0.03                                            ;
        2   -4.97939       /3/                                  ;
        4     0.8728     23.492    0.000          /D1b/         ;
        2    10.3055       /4/                                  ;
        3       0.03                                            ;
       18        0.2      0.844    8.000          /S1b/         ;
        3       0.03                                            ;
        5       0.33      9.846    8.000          /Q1b/         ;
                   9                                            ;
        3          0                                            ;
```

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   5.0    0.20434    9.895   8.000          /Q0/    ;
   3.0    0.1256                                    ;
   3.0    2.45                                      ;
  16.0    100        4.760   4.760                  ;
   5.0    0.33      -3.646   4.760          /F1a/   ;
 (lattice)
   9      5.000                                     ;
   5      0.330     -3.646   4.760          /F1b/   ;
   3      2.450                                     ;
   5      0.660      3.646   4.760          /F2/    ;
   3      2.450                                     ;
   5      0.330     -3.646   4.760          /F3b/   ;
  16      11.000     3.094                          ;
   6      106.000    0.250                  /SQ18/  ;
  16      11.000     3.131                          ;
   9                                                ;
   5.0    0.33      -3.646   4.760          /F1a/   ;
   3.0    2.45                                      ;
   3.0    0.1256                                    ;
  16.0    100        8.000   8.000                  ;
   5.0    0.20434    9.895   8.000                  ;
  16.0    11         3.094                          ;
   6.0    106        0.250                  /SQn/   ;
  16.0    11         3.131                          ;
   9.0                                              ;
  51.0    1         -5.000   5.000   0.200  /X2/    ;
  52.0    2        -20.000  20.000   1.000  /P2/    ;
  51.0    3         -5.000   5.000   0.200  /Y2/    ;
  52.0    4        -20.000  20.000   1.000  /YP2/   ;
  50.0    11         2.974   3.287   0.005  /P2/    ;
  51.0    101       -5.000   5.000   0.200  /X2/    ;
  52.0    102      -20.000  20.000   1.000  /MU2/   ;
  51.0    103       -5.000   5.000   0.200  /Y2/    ;
  52.0    104      -20.000  20.000   1.000  /YP2/   ;
  50.0    115       -1.000  -0.800   0.001  /PL2/   ;
( 50.0    5        -50.000  50.000   0.500 /Lout/   ;
 (end     turn           ,   repeat       n  times)
   (      3              )
   3      0                                 /TU3/   ;
 (begin   racetrack)
   9.0    2                                         ;
  16.0    11         3.094                          ;
   6.0    106        0.250                  /SQin/  ;
  16.0    11         3.131                          ;
      (beginning     180.000degree   double ac      )
   3      0                                         ;
   9      8                                         ;
  16      100        8.000   8.000                  ;
   5      0.33       9.895   8.000          /Q1a/   ;
   3      0.03                                      ;
  16      100        8.000   8.000                  ;
  18      0.2        0.844   8.000          /S1a/   ;
   3      0.03                                      ;
   2      10.3055    /1/                            ;
   4      0.8728    23.492   0.000          /D1a/   ;
   2      -4.97939   /2/                            ;
   3      0.03                                      ;
  18      0.2       -1.369   8.000          /S2a/   ;
   3      0.03                                      ;
   5      0.66      -9.895   8.000          /Q2a/   ;
   3      0.03                                      ;
  18      0.2       -1.369   8.000          /S2b/   ;
   3      0.03                                      ;
   2      -4.97939   /3/                            ;
   4      0.8728    23.492   0.000          /D1b/   ;
   2      10.3055    /4/                            ;
   3      0.03                                      ;
  18      0.2        0.844   8.000          /S1b/   ;
   3      0.03                                      ;
   5      0.33       9.846   8.000          /Q1b/   ;
          9                                         ;
   3      0                                         ;
   5.0    0.20434    9.895   8.000          /Q0/    ;
   3.0    0.1256                                    ;
   3.0    2.45                                      ;
  16.0    100        4.760   4.760                  ;
   5.0    0.33      -3.646   4.760          /F1a/   ;
 (lattice)
   9      5.000                                     ;
   5      0.330     -3.646   4.760          /F1b/   ;
   3      2.450                                     ;
   5      0.660      3.646   4.760          /F2/    ;
   3      2.450                                     ;
   5      0.330     -3.646   4.760          /F3b/   ;
  16      11.000     3.094                          ;
   6      106.000    0.250                  /SQ18/  ;
  16      11.000     3.131                          ;
   9                                                ;
   5.0    0.33      -3.646   4.760          /F1a/   ;
   3.0    2.45                                      ;
   3.0    0.1256                                    ;
  16.0    100        8.000   8.000                  ;
   5.0    0.20434    9.895   8.000                  ;
  16.0    11         3.094                          ;
   6.0    106        0.250                  /SQn/   ;
  16.0    11         3.131                          ;
   9.0                                              ;
( 51.0    1         -5.000   5.000   0.200  /X3/    ;
( 52.0    2        -20.000  20.000   1.000  /P3/    ;
( 51.0    3         -5.000   5.000   0.200  /Y3/    ;
( 52.0    4        -20.000  20.000   1.000  /YP3/   ;
( 50.0    11         2.974   3.287   0.005  /P3/    ;
( 51.0    101       -5.000   5.000   0.200  /X3/    ;
( 52.0    102      -20.000  20.000   1.000  /MU3/   ;
( 51.0    103       -5.000   5.000   0.200  /Y3/    ;
( 52.0    104      -20.000  20.000   1.000  /YP3/   ;
( 50.0    115       -1.000  -0.800   0.001  /PL3/   ;
( 50.0    5        -50.000  50.000   0.500 /Lout/   ;
 (end     turn           ,   repeat       n  times)
   (      4              )
   3      0                                 /TU4/   ;
 (begin   racetrack)
```

```
   9.0    2                                         ;
  16.0    11         3.094                          ;
   6.0    106        0.250                  /SQin/  ;
  16.0    11         3.131                          ;
      (beginning     180.000degree   double ac      )
   3      0                                         ;
   9      8                                         ;
  16      100        8.000   8.000                  ;
   5      0.33       9.895   8.000          /Q1a/   ;
   3      0.03                                      ;
  16      100        8.000   8.000                  ;
  18      0.2        0.844   8.000          /S1a/   ;
   3      0.03                                      ;
   2      10.3055    /1/                            ;
   4      0.8728    23.492   0.000          /D1a/   ;
   2      -4.97939   /2/                            ;
   3      0.03                                      ;
  18      0.2       -1.369   8.000          /S2a/   ;
   3      0.03                                      ;
   5      0.66      -9.895   8.000          /Q2a/   ;
   3      0.03                                      ;
  18      0.2       -1.369   8.000          /S2b/   ;
   3      0.03                                      ;
   2      -4.97939   /3/                            ;
   4      0.8728    23.492   0.000          /D1b/   ;
   2      10.3055    /4/                            ;
   3      0.03                                      ;
  18      0.2        0.844   8.000          /S1b/   ;
   3      0.03                                      ;
   5      0.33       9.846   8.000          /Q1b/   ;
          9                                         ;
   3      0                                         ;
   5.0    0.20434    9.895   8.000          /Q0/    ;
   3.0    0.1256                                    ;
   3.0    2.45                                      ;
) 16.0    100        4.760   4.760                  ;
   5.0    0.33      -3.646   4.760          /F1a/   ;
 (lattice)
   9      5.000                                     ;
   5      0.330     -3.646   4.760          /F1b/   ;
   3      2.450                                     ;
   5      0.660      3.646   4.760          /F2/    ;
   3      2.450                                     ;
   5      0.330     -3.646   4.760          /F3b/   ;
  16      11.000     3.094                          ;
   6      106.000    0.250                  /SQ18/  ;
  16      11.000     3.131                          ;
   9                                                ;
   5.0    0.33      -3.646   4.760          /F1a/   ;
   3.0    2.45                                      ;
   3.0    0.1256                                    ;
  16.0    100        8.000   8.000                  ;
   5.0    0.20434    9.895   8.000                  ;
  16.0    11         3.094                          ;
   6.0    106        0.250                  /SQn/   ;
  16.0    11         3.131                          ;
   9.0                                              ;
  51.0    1         -5.000   5.000   0.200  /X4/    ;
  52.0    2        -20.000  20.000   1.000  /P4/    ;
  51.0    3         -5.000   5.000   0.200  /Y4/    ;
  52.0    4        -20.000  20.000   1.000  /YP4/   ;
  50.0    11         2.974   3.287   0.005  /P4/    ;
  51.0    101       -5.000   5.000   0.200  /X4/    ;
  52.0    102      -20.000  20.000   1.000  /MU4/   ;
  51.0    103       -5.000   5.000   0.200  /Y4/    ;
  52.0    104      -20.000  20.000   1.000  /YP4/   ;
  50.0    115       -1.000  -0.800   0.001  /PL4/   ;
( 50.0    5        -50.000  50.000   0.500 /Lout/   ;   )
 (end     turn           ,   repeat       n  times)
   (      5              )
   3      0                                 /TU5/   ;
 (begin   racetrack)
   9.0    2                                         ;
  16.0    11         3.094                          ;
   6.0    106        0.250                  /SQin/  ;
  16.0    11         3.131                          ;
      (beginning     180.000degree   double ac      )
   3      0                                         ;
   9      8                                         ;
  16      100        8.000   8.000                  ;
   5      0.33       9.895   8.000          /Q1a/   ;
   3      0.03                                      ;
  16      100        8.000   8.000                  ;
  18      0.2        0.844   8.000          /S1a/   ;
   3      0.03                                      ;
   2      10.3055    /1/                            ;
   4      0.8728    23.492   0.000          /D1a/   ;
   2      -4.97939   /2/                            ;
   3      0.03                                      ;
  18      0.2       -1.369   8.000          /S2a/   ;
   3      0.03                                      ;
   5      0.66      -9.895   8.000          /Q2a/   ;
   3      0.03                                      ;
  18      0.2       -1.369   8.000          /S2b/   ;
   3      0.03                                      ;
   2      -4.97939   /3/                            ;
   4      0.8728    23.492   0.000          /D1b/   ;
)  2      10.3055    /4/                            ;
)  3      0.03                                      ;
) 18      0.2        0.844   8.000          /S1b/   ;
)  3      0.03                                      ;
)  5      0.33       9.846   8.000          /Q1b/   ;
)         9                                         ;
)  3      0                                         ;
)  5.0    0.20434    9.895   8.000          /Q0/    ;
)  3.0    0.1256                                    ;
)  3.0    2.45                                      ;
) 16.0    100        4.760   4.760                  ;
   5.0    0.33      -3.646   4.760          /F1a/   ;
 (lattice)
   9      5.000                                     ;
   5      0.330     -3.646   4.760          /F1b/   ;
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```
         3      2.450                                        ;
         5      0.660     3.646    4.760          /F2/       ;
         3      2.450                                        ;
         5      0.330    -3.646    4.760          /F3b/      ;
        16     11.000     3.094                              ;
         6    106.000     0.250                   /SQ18/     ;
        16     11.000     3.131                              ;
         9                                                   ;
       5.0       0.33    -3.646    4.760          /F1a/      ;
       3.0       2.45                                        ;
       3.0     0.1256                                        ;
      16.0        100     8.000    8.000                     ;
       5.0    0.20434     9.895    8.000                     ;
      16.0         11     3.094                              ;
       6.0        106     0.250                   /SQn/      ;
      16.0         11     3.131                              ;
       9.0                                                   ;
(     51.0          1    -5.000    5.000   0.200  /X5/       ;
(     52.0          2   -20.000   20.000   0.500  /P5/       ;
(     51.0          3    -5.000    5.000   0.200  /Y5/       ;
(     52.0          4   -20.000   20.000   0.500  /YP5/      ;
(     50.0         11     2.974    3.287   0.005  /P5/       ;
(     51.0        101    -5.000    5.000   0.200  /X5/       ;
(     52.0        102   -20.000   20.000   0.500  /MU5/      ;
(     51.0        103    -5.000    5.000   0.200  /Y5/       ;
(     52.0        104   -20.000   20.000   0.500  /YP5/      ;
(     50.0        115    -1.000   -0.800   0.001  /PL5/      ;
(     50.0          5   -50.000   50.000   0.500 /Lout/      ;
      (end       turn         ,   repeat       n  times)
        (           6         )
         3          0                             /TU6/      ;
   (begin   racetrack)
       9.0          2                                        ;
      16.0         11     3.094                              ;
       6.0        106     0.250                   /SQin/     ;
      16.0         11     3.131                              ;
        (beginning       180.000degree    double ac          )
         3          0                                        ;
         9          8                                        ;
        16        100     8.000    8.000                     ;
         5       0.33     9.895    8.000          /Q1a/      ;
         3       0.03                                        ;
        16        100     8.000    8.000                     ;
        18        0.2     0.844    8.000          /S1a/      ;
         3       0.03                                        ;
         2    10.3055      /1/                               ;
         4     0.8728    23.492    0.000          /D1a/      ;
         2   -4.97939      /2/                               ;
         3       0.03                                        ;
        18        0.2    -1.369    8.000          /S2a/      ;
         3       0.03                                        ;
         5       0.66    -9.895    8.000          /Q2a/      ;
         3       0.03                                        ;
        18        0.2    -1.369    8.000          /S2b/      ;
         3       0.03                                        ;
         2   -4.97939      /3/                               ;
         4     0.8728    23.492    0.000          /D1b/      ;
         2    10.3055      /4/                               ;
         3       0.03                                        ;
        18        0.2     0.844    8.000          /S1b/      ;
         3       0.03                                        ;
         5       0.33     9.846    8.000          /Q1b/      ;
                    9                                        ;
         3          0                                        ;
       5.0    0.20434     9.895    8.000          /Q0/       ;
       3.0     0.1256                                        ;
       3.0       2.45                                        ;
      16.0        100     4.760    4.760                     ;
       5.0       0.33    -3.646    4.760          /F1a/      ;
  (lattice)
         9      5.000                                        ;
         5      0.330    -3.646    4.760          /F1b/      ;
         3      2.450                                        ;
         5      0.660     3.646    4.760          /F2/       ;
         3      2.450                                        ;
         5      0.330    -3.646    4.760          /F3b/      ;
        16     11.000     3.094                              ;
         6    106.000     0.250                   /SQ18/     ;
        16     11.000     3.131                              ;
         9                                                   ;
       5.0       0.33    -3.646    4.760          /F1a/      ;
       3.0       2.45                                        ;
       3.0     0.1256                                        ;
      16.0        100     8.000    8.000                     ;
       5.0    0.20434     9.895    8.000                     ;
      16.0         11     3.094                              ;
       6.0        106     0.250                   /SQn/      ;
      16.0         11     3.131                              ;
       9.0                                                   ;
      51.0          1    -5.000    5.000   0.200  /X6/       ;
      52.0          2   -20.000   20.000   1.000  /P6/       ;
      51.0          3    -5.000    5.000   0.200  /Y6/       ;
      52.0          4   -20.000   20.000   1.000  /YP6/      ;
      50.0         11     2.974    3.287   0.005  /P6/       ;
      51.0        101    -5.000    5.000   0.200  /X6/       ;
      52.0        102   -20.000   20.000   1.000  /MU6/      ;
      51.0        103    -5.000    5.000   0.200  /Y6/       ;
      52.0        104   -20.000   20.000   1.000  /YP6/      ;
      50.0        115    -1.000   -0.800   0.001  /PL6/      ;
(     50.0          5   -50.000   50.000   0.500 /Lout/      ;
      (end       turn         ,   repeat       n  times)
        (           7         )
         3          0                             /TU7/      ;
   (begin   racetrack)
       9.0          2                                        ;
      16.0         11     3.094                              ;
       6.0        106     0.250                   /SQin/     ;
      16.0         11     3.131                              ;
        (beginning       180.000degree    double ac          )
         3          0                                        ;
         9          8                                        ;
        16        100     8.000    8.000                     ;
```

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         5       0.33     9.895    8.000          /Q1a/      ;
         3       0.03                                        ;
        16        100     8.000    8.000                     ;
        18        0.2     0.844    8.000          /S1a/      ;
         3       0.03                                        ;
         2    10.3055      /1/                               ;
         4     0.8728    23.492    0.000          /D1a/      ;
         2   -4.97939      /2/                               ;
         3       0.03                                        ;
        18        0.2    -1.369    8.000          /S2a/      ;
         3       0.03                                        ;
         5       0.66    -9.895    8.000          /Q2a/      ;
         3       0.03                                        ;
        18        0.2    -1.369    8.000          /S2b/      ;
         3       0.03                                        ;
         2   -4.97939      /3/                               ;
         4     0.8728    23.492    0.000          /D1b/      ;
)        2    10.3055      /4/                               ;
)        3       0.03                                        ;
)       18        0.2     0.844    8.000          /S1b/      ;
)        3       0.03                                        ;
)        5       0.33     9.846    8.000          /Q1b/      ;
)                   9                                        ;
)        3          0                                        ;
)      5.0    0.20434     9.895    8.000          /Q0/       ;
)      3.0     0.1256                                        ;
)      3.0       2.45                                        ;
)     16.0        100     4.760    4.760                     ;
       5.0       0.33    -3.646    4.760          /F1a/      ;
  (lattice)
         9      5.000                                        ;
         5      0.330    -3.646    4.760          /F1b/      ;
         3      2.450                                        ;
         5      0.660     3.646    4.760          /F2/       ;
         3      2.450                                        ;
         5      0.330    -3.646    4.760          /F3b/      ;
        16     11.000     3.094                              ;
         6    106.000     0.250                   /SQ18/     ;
        16     11.000     3.131                              ;
         9                                                   ;
       5.0       0.33    -3.646    4.760          /F1a/      ;
       3.0       2.45                                        ;
       3.0     0.1256                                        ;
      16.0        100     8.000    8.000                     ;
       5.0    0.20434     9.895    8.000                     ;
      16.0         11     3.094                              ;
       6.0        106     0.250                   /SQn/      ;
      16.0         11     3.131                              ;
       9.0                                                   ;
(     51.0          1    -5.000    5.000   0.200  /X7/       ;     )
(     52.0          2   -20.000   20.000   1.000  /P7/       ;     )
(     51.0          3    -5.000    5.000   0.200  /Y7/       ;     )
(     52.0          4   -20.000   20.000   1.000  /YP7/      ;     )
(     50.0         11     2.974    3.287   0.005  /P7/       ;     )
(     51.0        101    -5.000    5.000   0.200  /X7/       ;     )
(     52.0        102   -20.000   20.000   1.000  /MU7/      ;     )
(     51.0        103    -5.000    5.000   0.200  /Y7/       ;     )
(     52.0        104   -20.000   20.000   1.000  /YP7/      ;     )
(     50.0        115    -1.000   -0.800   0.001  /PL7/      ;     )
(     50.0        105   -50.000   50.000   0.500 /Lout/      ;     )
      (end       turn         ,   repeat       n  times)
        (           8         )
         3          0                             /TU8/      ;
   (begin   racetrack)
       9.0          2                                        ;
      16.0         11     3.094                              ;
       6.0        106     0.250                   /SQin/     ;
      16.0         11     3.131                              ;
        (beginning       180.000degree    double ac          )
         3          0                                        ;
         9          8                                        ;
        16        100     8.000    8.000                     ;
         5       0.33     9.895    8.000          /Q1a/      ;
         3       0.03                                        ;
        16        100     8.000    8.000                     ;
        18        0.2     0.844    8.000          /S1a/      ;
         3       0.03                                        ;
         2    10.3055      /1/                               ;
         4     0.8728    23.492    0.000          /D1a/      ;
         2   -4.97939      /2/                               ;
         3       0.03                                        ;
        18        0.2    -1.369    8.000          /S2a/      ;
         3       0.03                                        ;
         5       0.66    -9.895    8.000          /Q2a/      ;
         3       0.03                                        ;
        18        0.2    -1.369    8.000          /S2b/      ;
         3       0.03                                        ;
         2   -4.97939      /3/                               ;
         4     0.8728    23.492    0.000          /D1b/      ;
         2    10.3055      /4/                               ;
         3       0.03                                        ;
        18        0.2     0.844    8.000          /S1b/      ;
         3       0.03                                        ;
         5       0.33     9.846    8.000          /Q1b/      ;
                    9                                        ;
         3          0                                        ;
       5.0    0.20434     9.895    8.000          /Q0/       ;
       3.0     0.1256                                        ;
       3.0       2.45                                        ;
)     16.0        100     4.760    4.760                     ;
       5.0       0.33    -3.646    4.760          /F1a/      ;
  (lattice)
         9      5.000                                        ;
         5      0.330    -3.646    4.760          /F1b/      ;
         3      2.450                                        ;
         5      0.660     3.646    4.760          /F2/       ;
         3      2.450                                        ;
         5      0.330    -3.646    4.760          /F3b/      ;
        16     11.000     3.094                              ;
         6    106.000     0.250                   /SQ18/     ;
        16     11.000     3.131                              ;
         9                                                   ;
```

```
      5.0        0.33        -3.646     4.760          /F1a/          ;
      3.0        2.45                                                 ;
      3.0      0.1256                                                 ;
     16.0         100         8.000     8.000                         ;
      5.0     0.20434         9.895     8.000                         ;
     16.0          11         3.094                                   ;
      6.0         106         0.250                     /SQn/         ;
     16.0          11         3.131                                   ;
      9.0                                                             ;
(    51.0           1        -5.000     5.000    0.200   /X8/         ;        )
(    52.0           2       -20.000    20.000    1.000   /P8/         ;        )
(    51.0           3        -5.000     5.000    0.200   /Y8/         ;        )
(    52.0           4       -20.000    20.000    1.000  /YP8/         ;        )
(    50.0          11         2.974     3.287    0.005   /P8/         ;        )
     51.0         101        -5.000     5.000    0.200   /X8/         ;
     52.0         102       -20.000    20.000    1.000  /MU8/         ;
     51.0         103        -5.000     5.000    0.200   /Y8/         ;
     52.0         104       -20.000    20.000    1.000  /YP8/         ;
     50.0         115        -1.000    -0.800    0.001  /PL8/         ;
(    50.0           5       -50.000    50.000    0.500 /Lout/         ;        )
    (end        turn              ,   repeat        n  times)

  SENTINEL
  SENTINEL
```
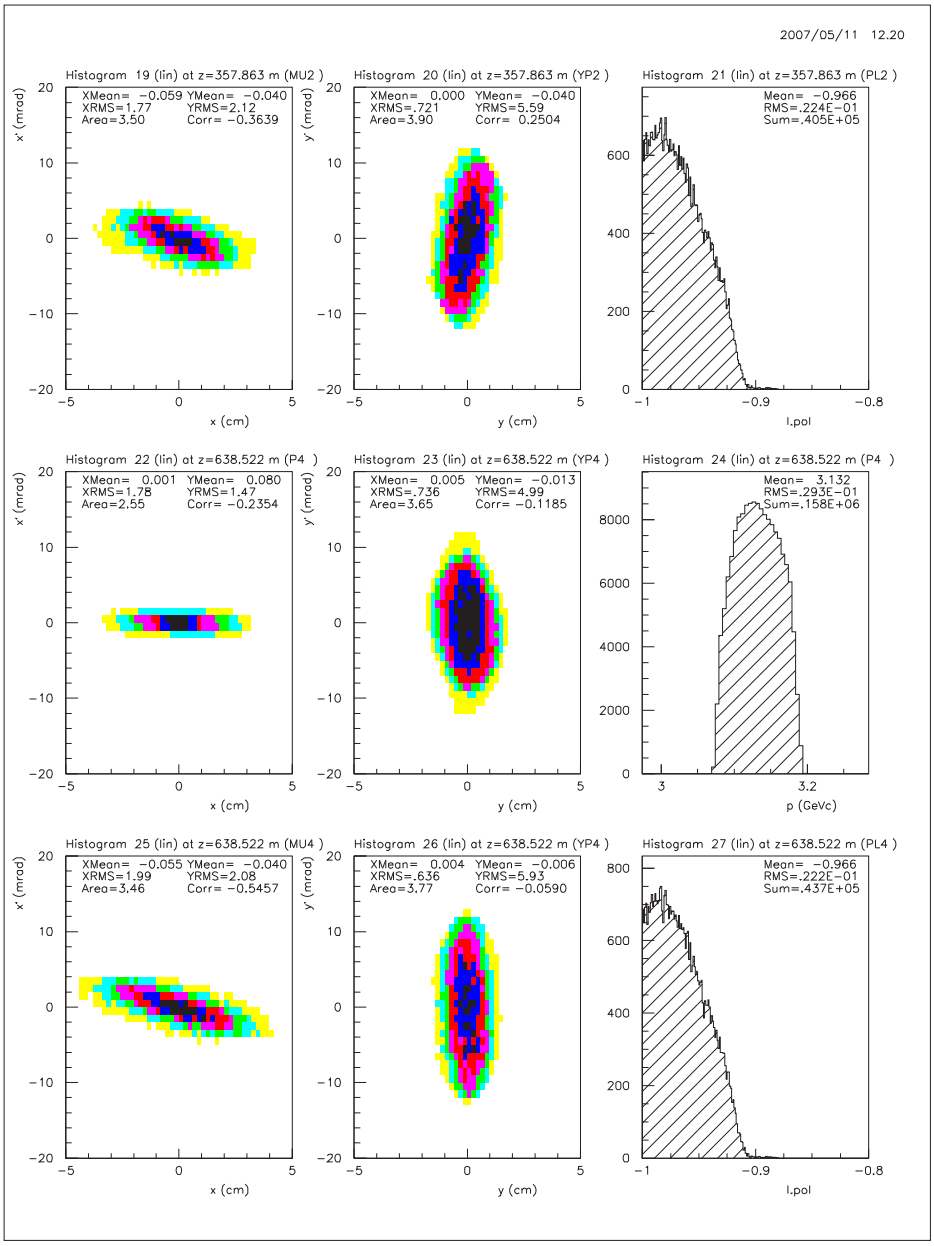
Figure 22: Muon accumulator ring